Carte universitară

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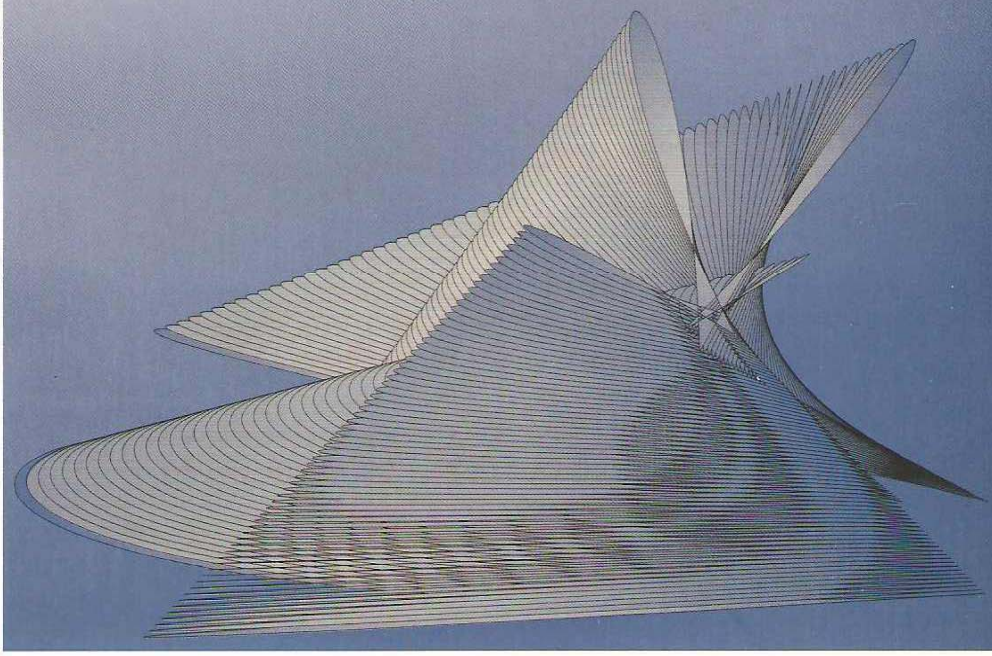
MECANICĂ

CULEGERE DE PROBLEME DATE LA EXAMEN



CONSPRESS BUCUREŞTI

PREFAȚĂ

Lucrarea conține 342 de probleme care au constituit subiecte de examen, în peste 15 ani universitari (peste 45 de sesiuni de examene), la disciplina MECANICA TEORETICĂ, date studenților de la facultatea de Construcții Civile, Industriale și Agricole și la Departamentul de Inginerie Civilă în limba engleză din cadrul Universității de Construcții din București la cele două examene : Mecanică I și Mecanică II, acolo unde autorii își desfășoară activitatea didactică. În consecință ea se prezintă ca o culegere de probleme caracteristice aprofundării acestei discipline importante pentru pregătirea unui viitor inginer constructor și necesare verificării cunoștințelor dobândite în cadrul cursului și seminarului de mecanică.

Lucrarea are două părți : o primă parte conținând enunțurile problemelor (opt capitole din care primele patru se referă la examenul de Mecanică I, iar următoarele patru la examenul de Mecanică II) și a doua parte prezentând rezultatele rezolvărilor acestora. Culegerea de probleme are intenția de a fi un instrument în procesul de seminarizare și de studiul individual în pregătirea studenților , viitori ingineri constructori, pentru examenele de mecanică. Fiecare capitol din lucrare reprezintă un anumit tip de problemă. Lucrarea poate fi folositoare și altor studenți, sau ingineri cu pregătire tehnică.

Autorii mulțumesc anticipat celor care vor face observații și sugestii pe marginea celor prezentate în lucrare.

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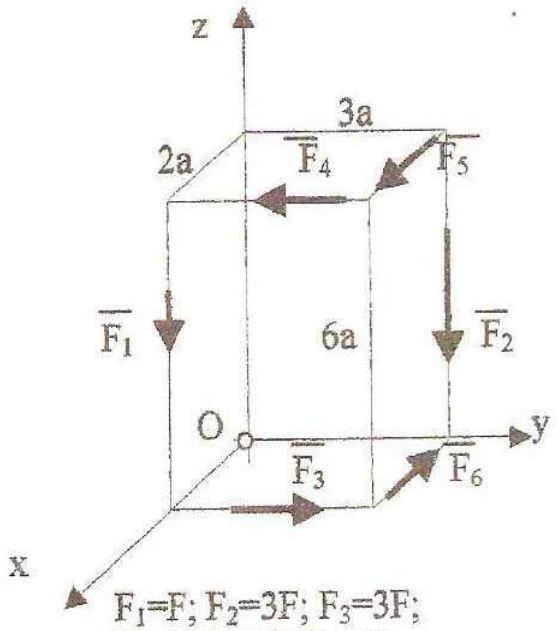
ENUNȚURI

A. MECANICA I

CAPITOLUL I. REDUCEREA SISTEMELOR DE FORTE OARECARE

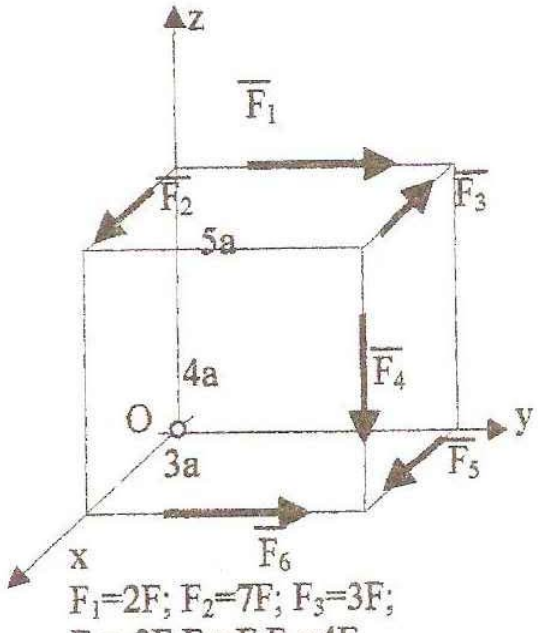
Problemele I.1.- I.40.

Să se calculeze torsorul în punctul O al sistemului de forţe reprezentat în figurile 1- 40 şi să se reprezinte cel mai simplu sistem echivalent.



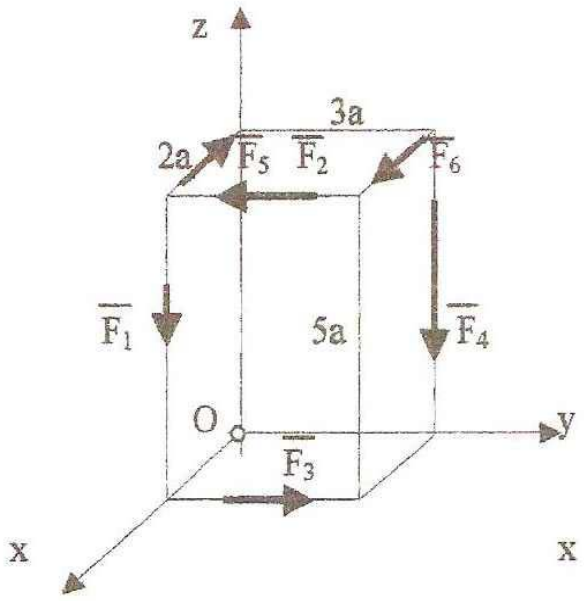
$F_1=F; F_2=3F; F_3=3F;$
$F_4= F; F_5=2F; F_6=4F$

Fig.I.1.



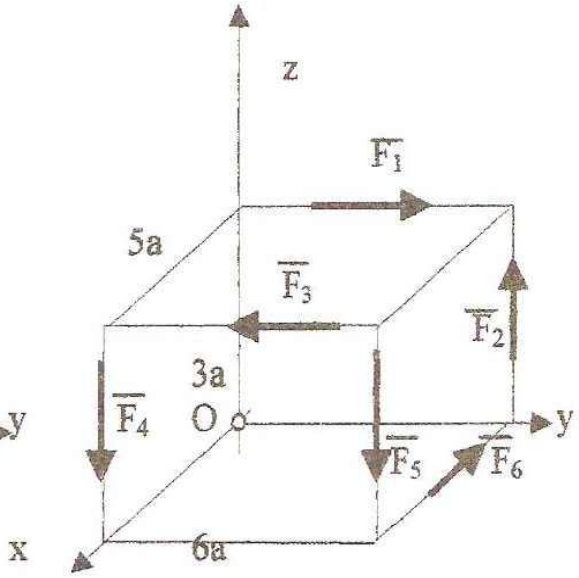
$F_1=2F; F_2=7F; F_3=3F;$
$F_4= 3F; F_5=F; F_6=4F$

Fig.I.2.



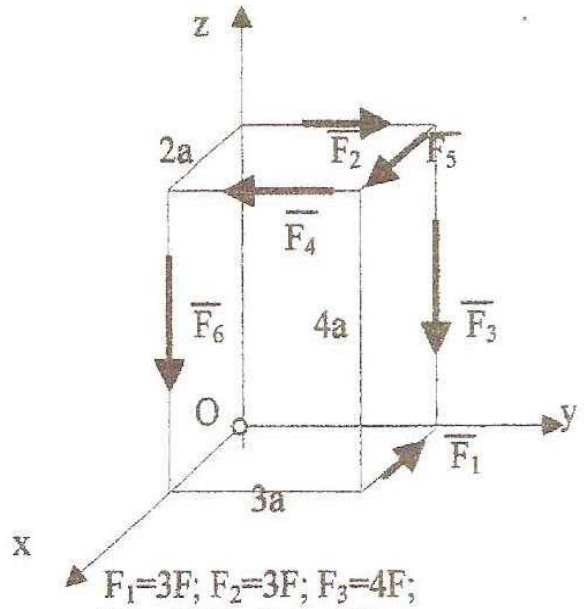
$F_1=F$; $F_2=2F$; $F_3=3F$;
$F_4=F$; $F_5=4F$; $F_6=5F$

Fig.I.3.



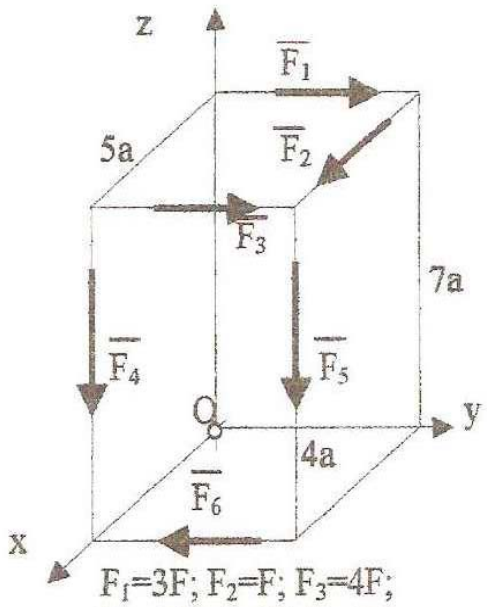
$F_1=3F$; $F_2=4F$; $F_3=3F$;
$F_4=2F$; $F_5=6F$; $F_6=5F$

Fig.I.4.



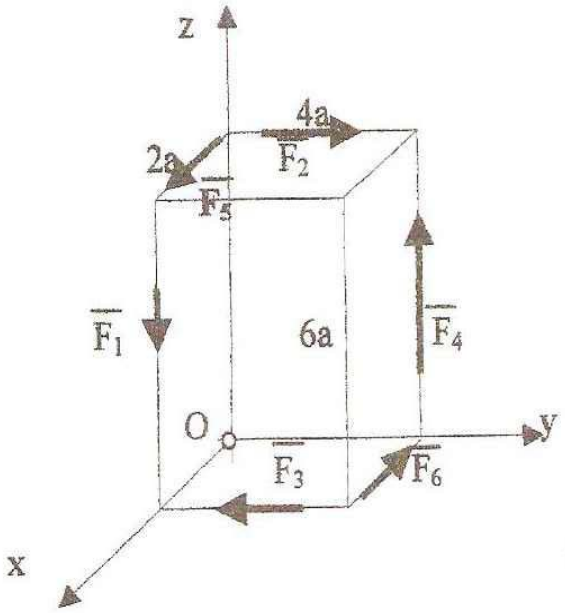
$F_1=3F$; $F_2=3F$; $F_3=4F$;
$F_4=5F$; $F_5=F$; $F_6=2F$

Fig.I.5.



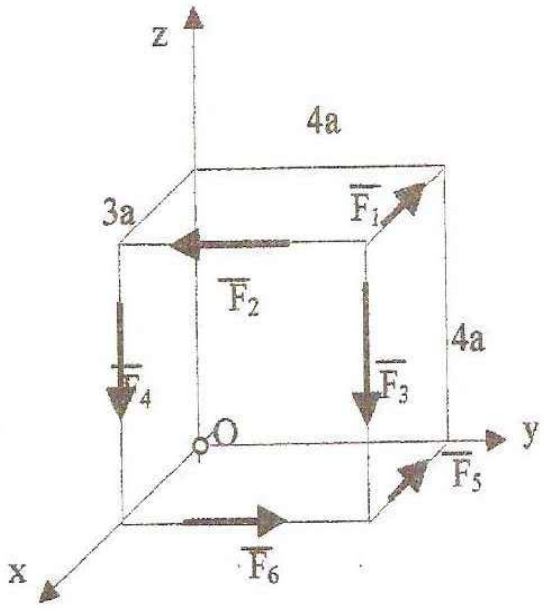
$F_1=3F$; $F_2=F$; $F_3=4F$;
$F_4=2F$; $F_5=5F$; $F_6=3F$

Fig.I.6.



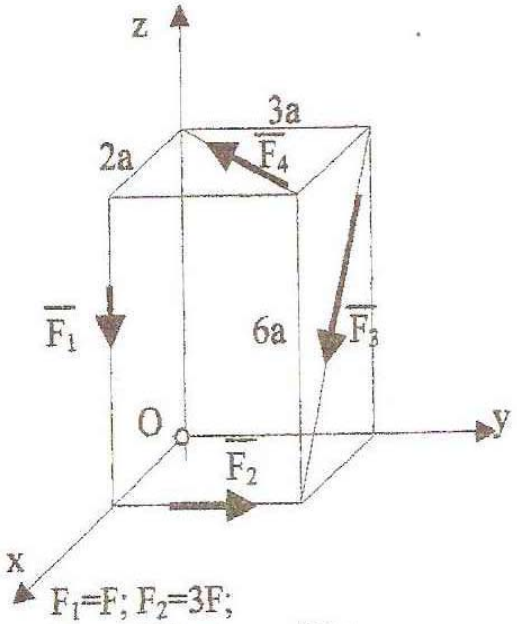
$F_1=F$; $F_2=2F$; $F_3=2F$;
$F_4=3F$; $F_5=4F$; $F_6=5F$

Fig.I.7.



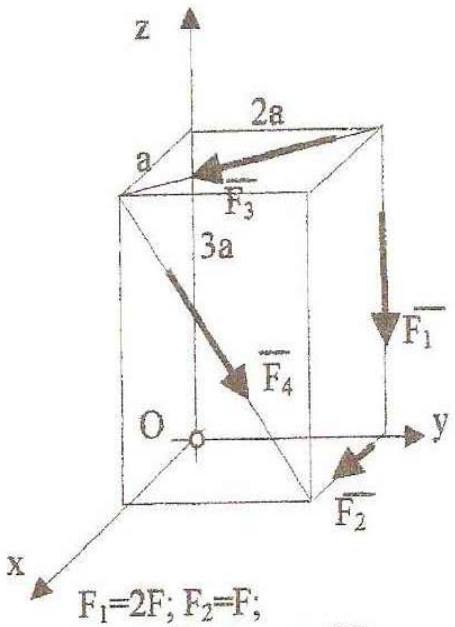
$F_1=3F$; $F_2=2F$; $F_3=3F$;
$F_4=4F$; $F_5=F$; $F_6=3F$

Fig.I.8.



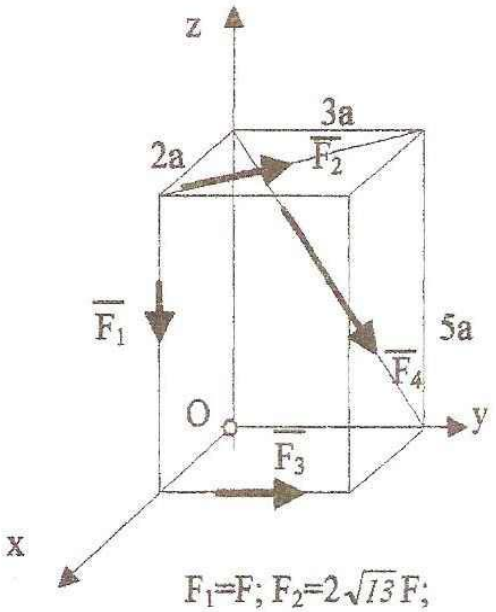
$F_1=F$; $F_2=3F$;
$F_3=3\sqrt{10}\,F$; $F_4=\sqrt{13}\,F$

Fig.I.9.



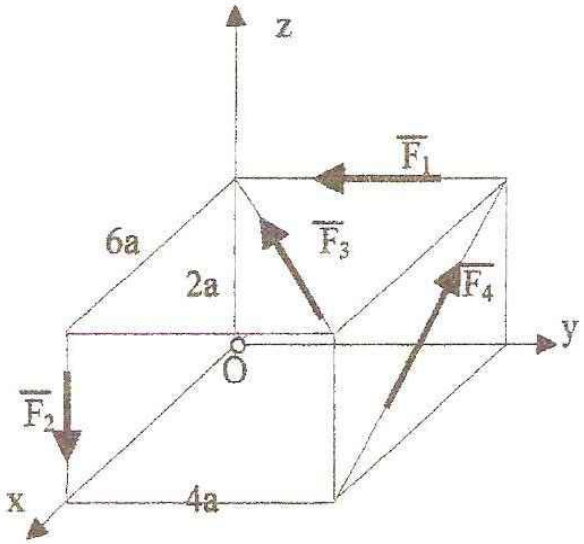
$F_1=2F$; $F_2=F$;
$F_3=2\sqrt{5}\,F$; $F_4=2\sqrt{13}\,F$

Fig.I.10.



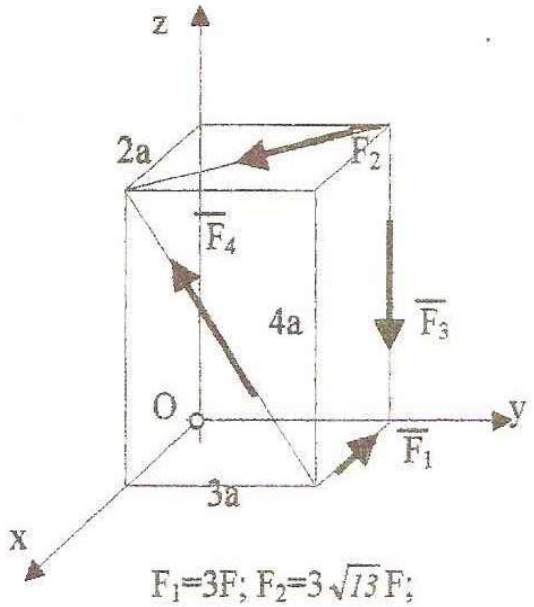
$F_1=F$; $F_2=2\sqrt{13}\,F$;
$F_3=3F$; $F_4=\sqrt{34}\,F$

Fig.I.11.



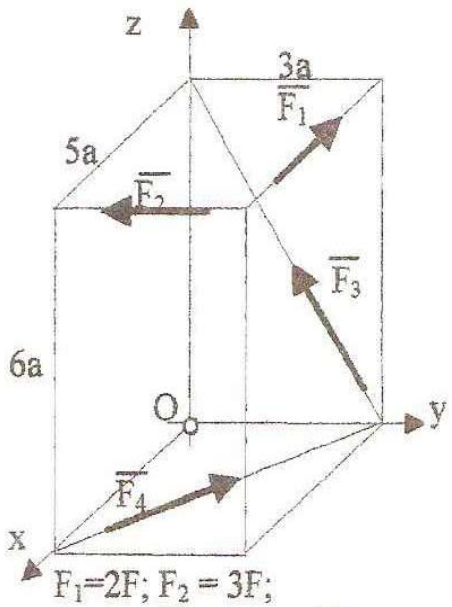
$F_1=3F$; $F_2=4F$;
$F_3=2\sqrt{13}\,F$; $F_4=\sqrt{10}\,F$

Fig.I.12.



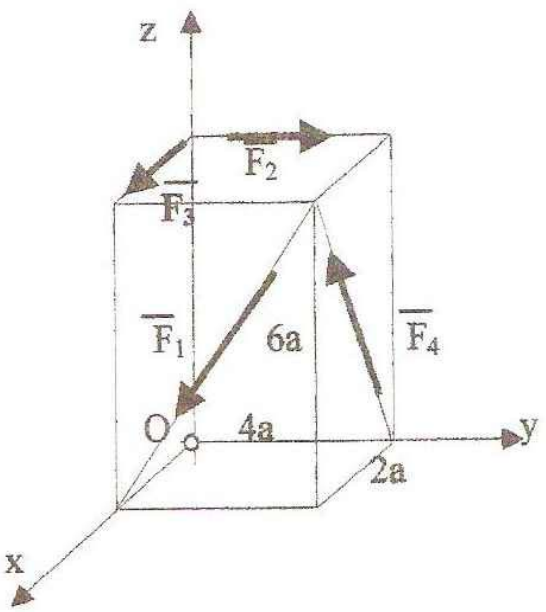
$F_1=3F$; $F_2=3\sqrt{13}\,F$;
$F_3=4F$; $F_4=10F$

Fig.I.13.



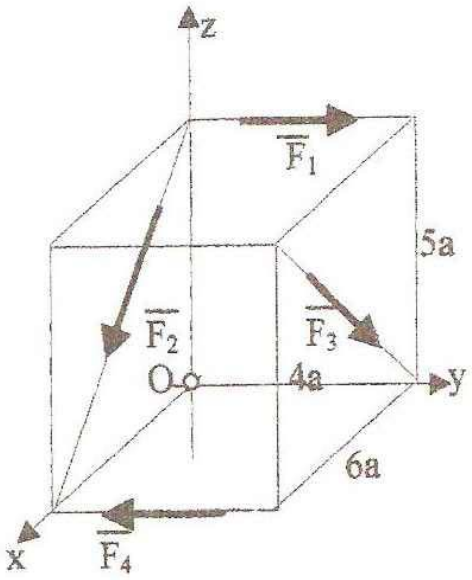
$F_1=2F$; $F_2=3F$;
$F_3=2\sqrt{5}\,F$; $F_4=\sqrt{61}\,F$.

Fig.I.14.



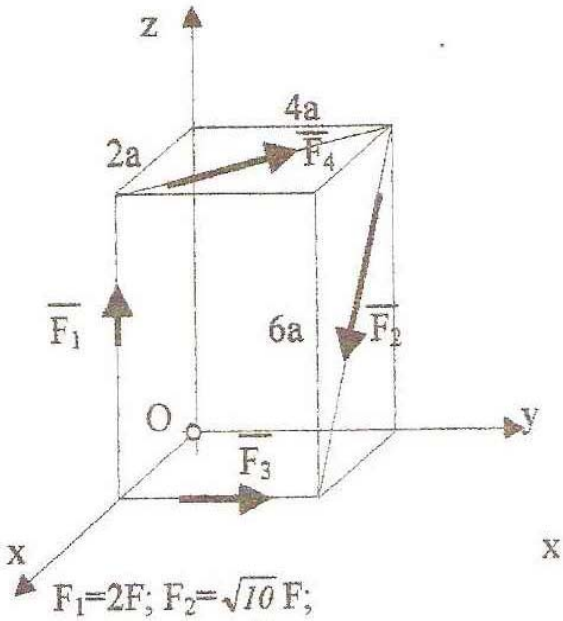
$F_1=\sqrt{13}\,F$; $F_2=2F$;
$F_3=2F$; $F_4=3\sqrt{10}\,F$

Fig.I.15.



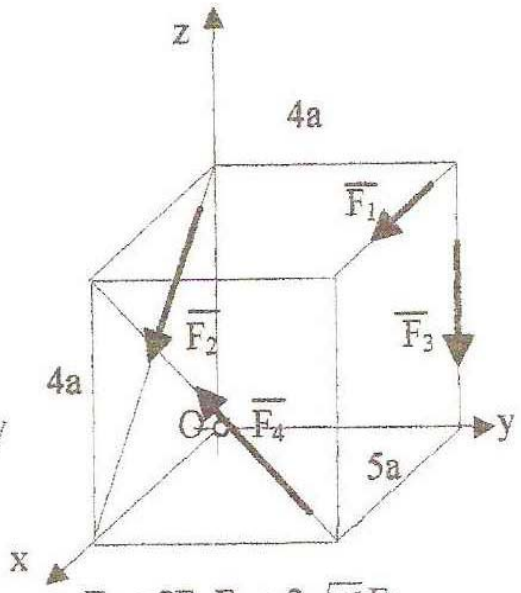
$F_1=2F$; $F_2=\sqrt{61}\,F$;
$F_3=2\sqrt{61}\,F$; $F_4=4F$.

Fig.I.16.



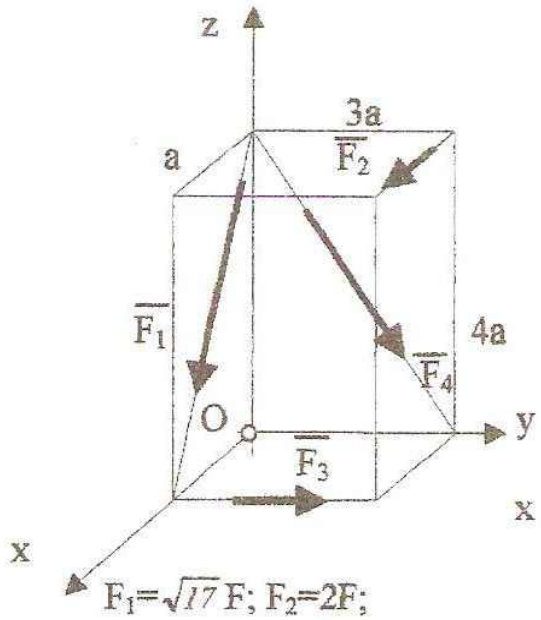
$F_1=2F$; $F_2=\sqrt{10}\,F$;
$F_3=3F$; $F_4=\sqrt{5}\,F$

Fig.I.17.



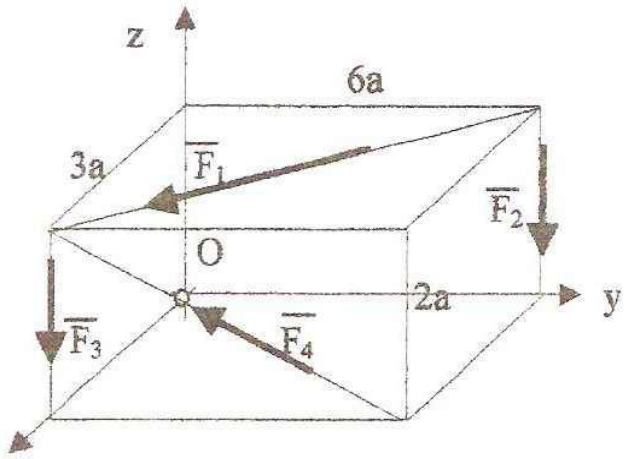
$F_1=2F$; $F_2=2\sqrt{41}\,F$;
$F_3=4F$; $F_4=3\sqrt{2}\,F$

Fig.I.18.



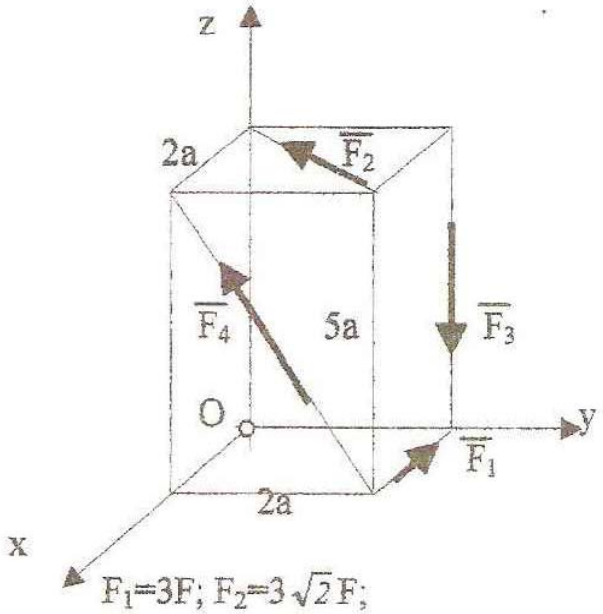
$F_1=\sqrt{17}\,F$; $F_2=2F$;
$F_3=3F$; $F_4=10F$

Fig.I.19.



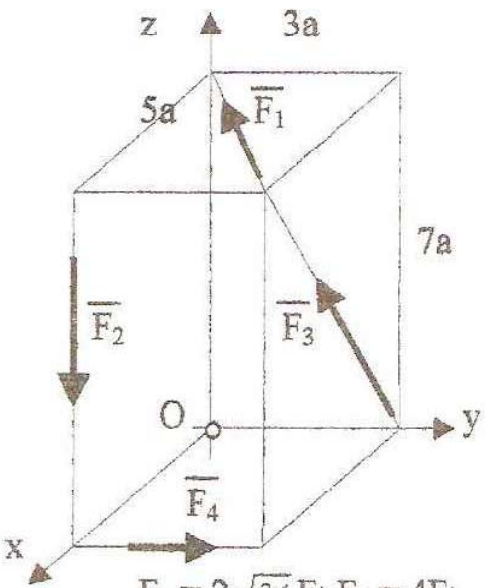
$F_1=4\sqrt{5}\,F$; $F_2=2F$;
$F_3=3F$; $F_4=4\sqrt{10}\,F$

Fig.I.20.



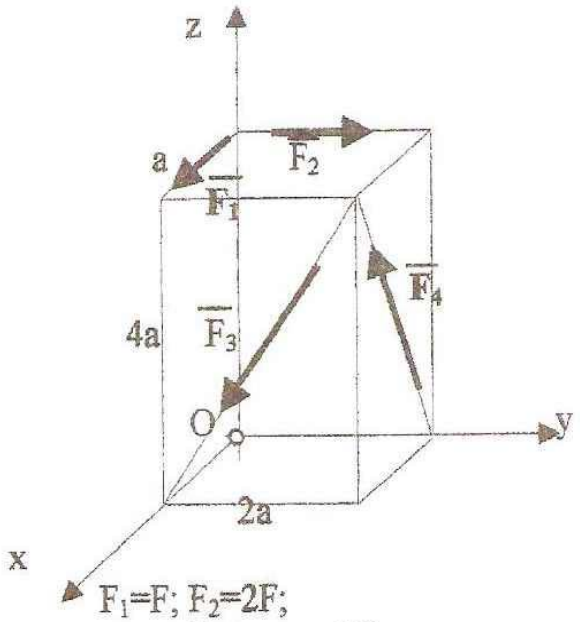
$F_1=3F$; $F_2=3\sqrt{2}\,F$;
$F_3=4F$; $F_4=\sqrt{29}\,F$

Fig.I.21.



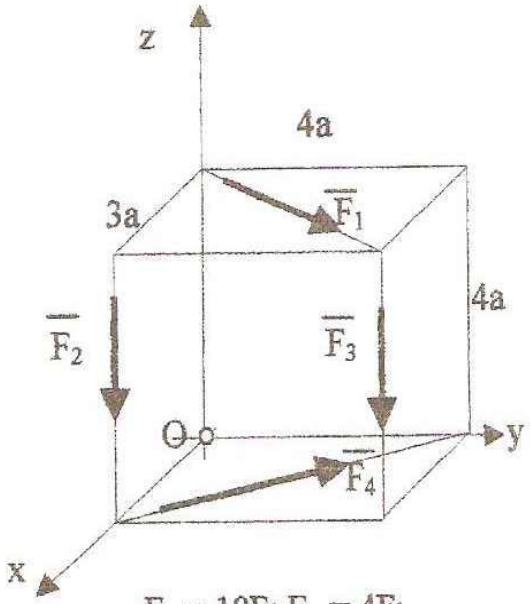
$F_1=2\sqrt{34}\,F$; $F_2=4F$;
$F_3=\sqrt{74}\,F$; $F_4=5F$

Fig.I.22.



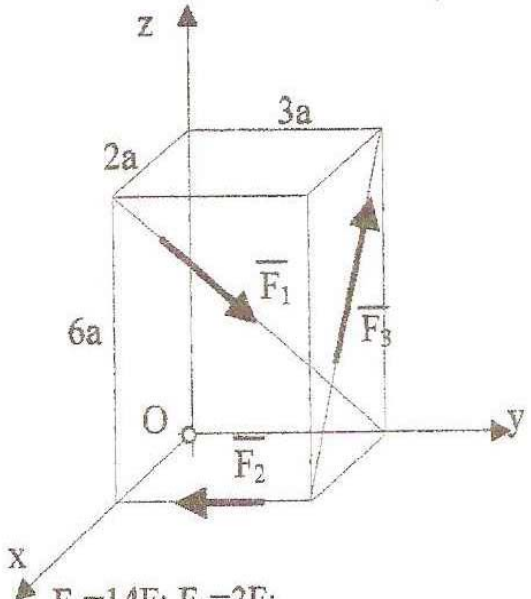
$F_1=F$; $F_2=2F$;
$F_3=2\sqrt{5}\,F$; $F_4=\sqrt{17}\,F$

Fig.I.23.



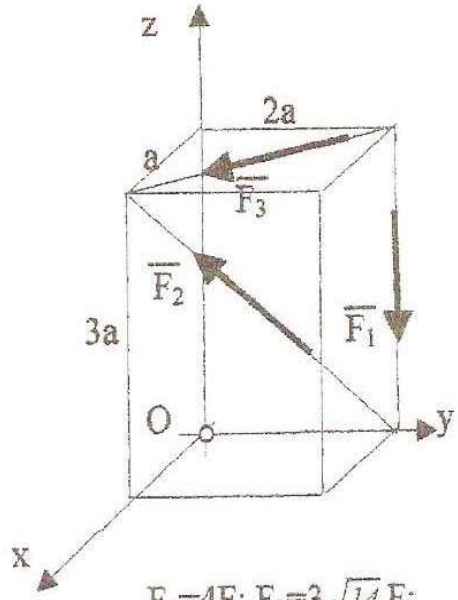
$F_1 = 10F$; $F_2 = 4F$;
$F_3 = 3F$; $F_4 = 15F$

Fig.I.24.



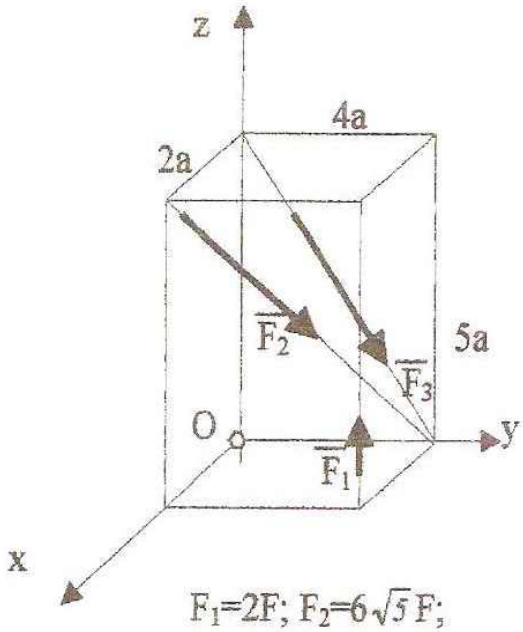
$F_1=14F$; $F_2=3F$;
$F_3=2\sqrt{10}\,F$

Fig.I.25.



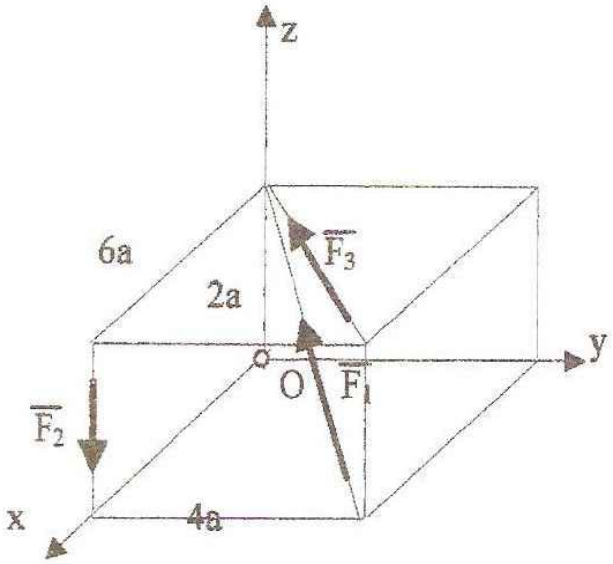
$F_1=4F$; $F_2=3\sqrt{14}\,F$;
$F_3=2\sqrt{5}\,F$;

Fig.I.26.



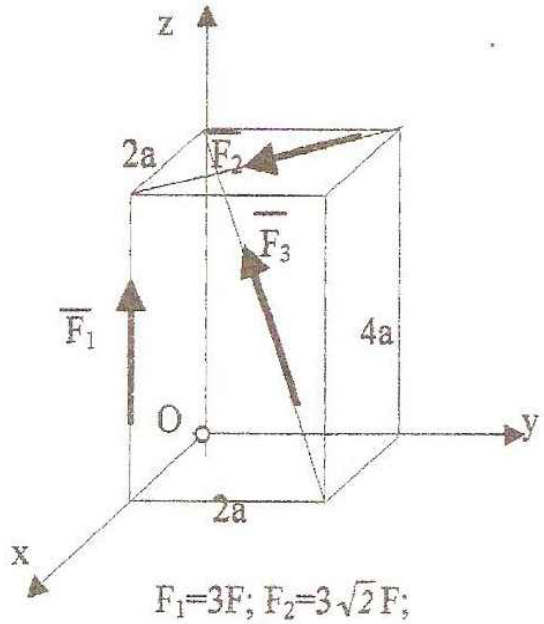
$F_1=2F$; $F_2=6\sqrt{5}\,F$;
$F_3=3\sqrt{41}\,F$;

Fig.I.27.



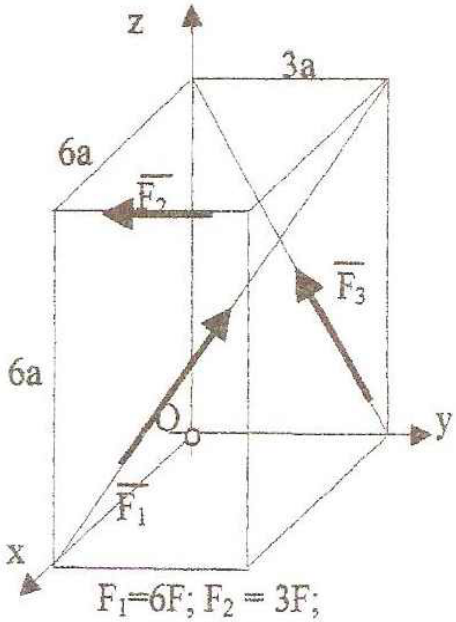
$F_1=3\sqrt{14}\,F$; $F_2=4F$;
$F_3=2\sqrt{13}\,F$

Fig.I.28.



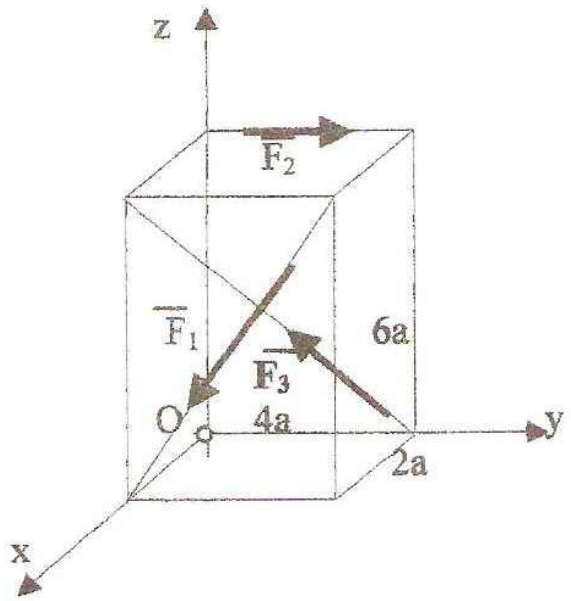
$F_1=3F$; $F_2=3\sqrt{2}\,F$;
$F_3=4\sqrt{6}\,F$;

Fig.I.29.



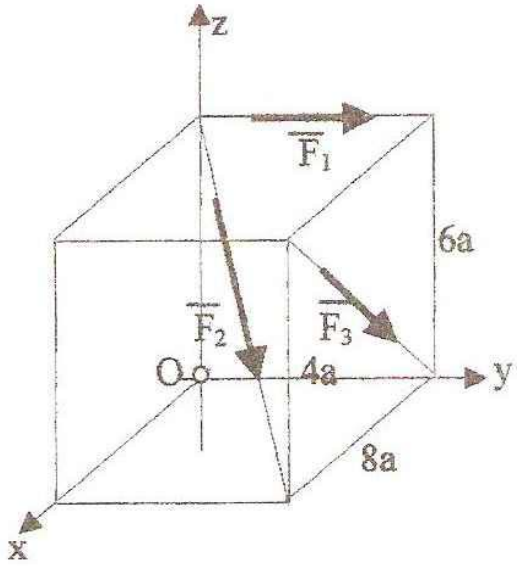
$F_1=6F$; $F_2=3F$;
$F_3=2\sqrt{5}\,F$

Fig.I.30.



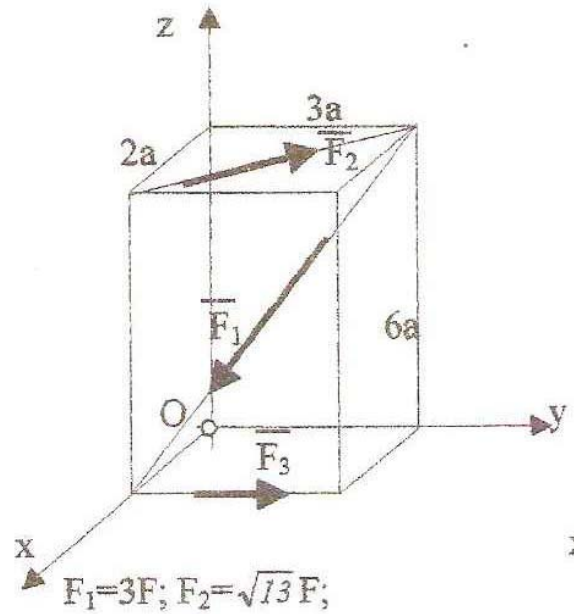
$F_1=2\sqrt{13}\,F$; $F_2=3F$;
$F_3=4\sqrt{14}\,F$;

Fig.I.31.



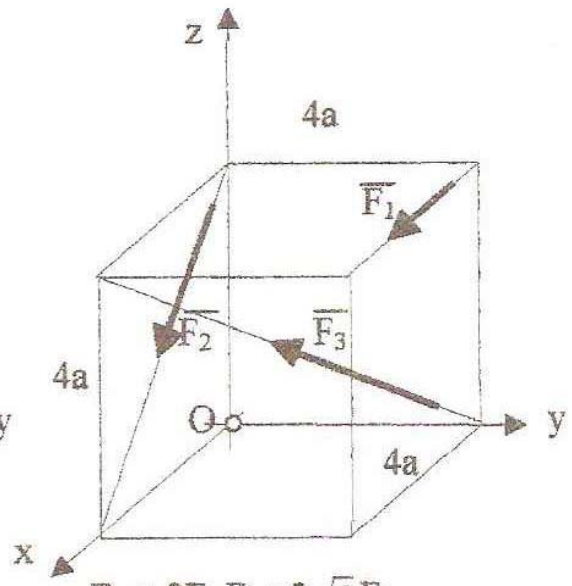
$F_1 = 2F$; $F_2 = 3\sqrt{29}\,F$;
$F_3 = 10F$

Fig.I.32.



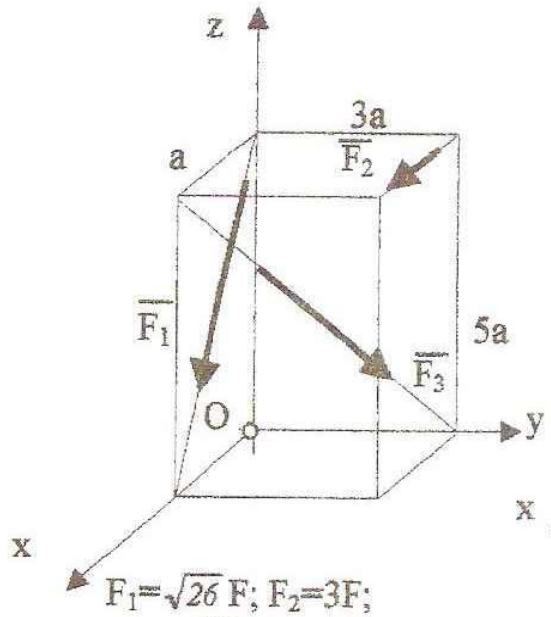
$F_1=3F$; $F_2=\sqrt{13}\,F$;
$F_3=14F$

Fig.I.33.



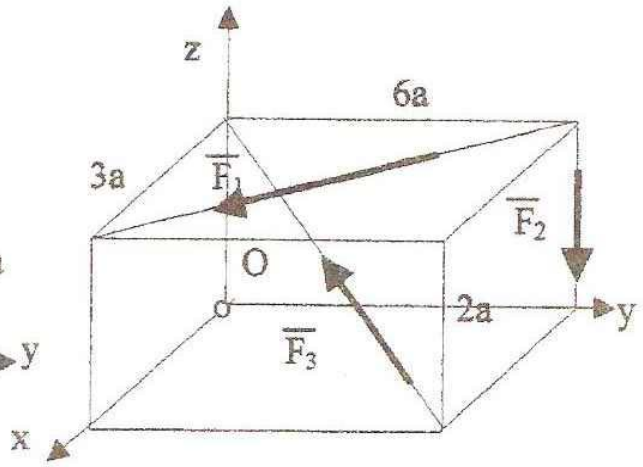
$F_1 = 2F$; $F_2 = 3\sqrt{2}\,F$;
$F_3 = 4\sqrt{3}\,F$

Fig.I.34.



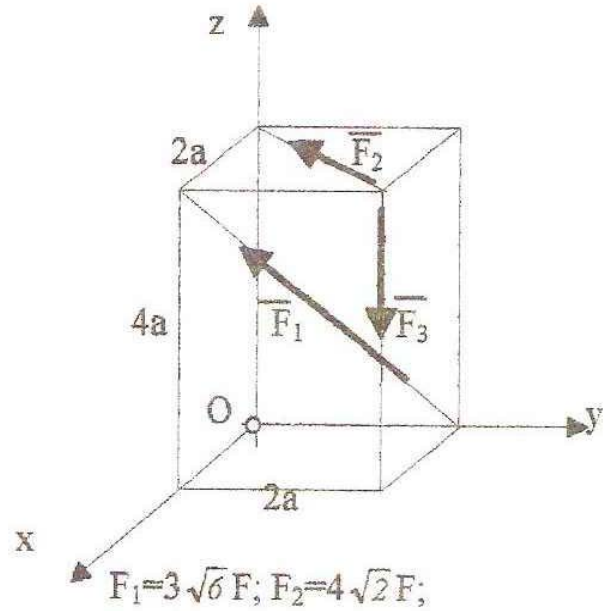
$F_1=\sqrt{26}\,F;\ F_2=3F;$
$F_3=3\sqrt{35}\,F$

Fig.I.35.



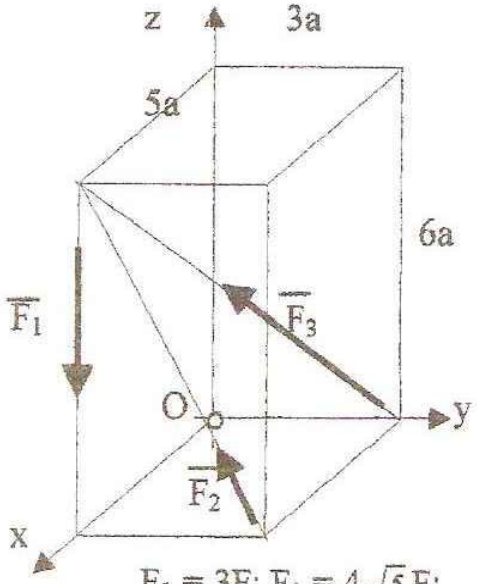
$F_1=2\sqrt{5}\,F;\ F_2=3F;$
$F_3=7F$

Fig.I.36.



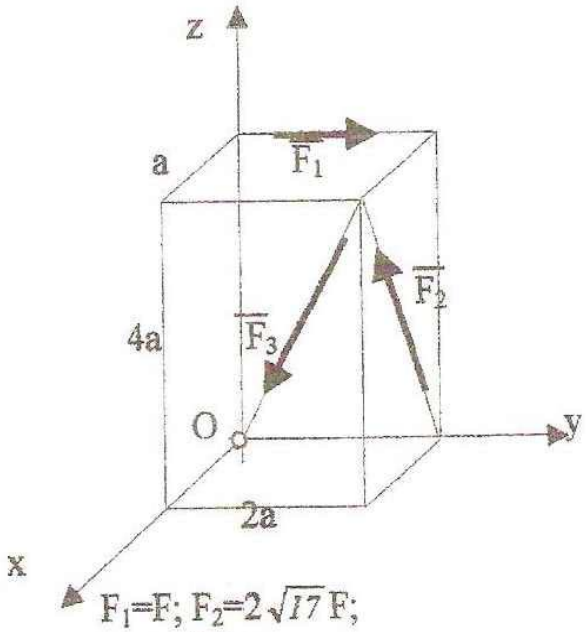
$F_1=3\sqrt{6}\,F;\ F_2=4\sqrt{2}\,F;$
$F_3=4F$

Fig.I.37.



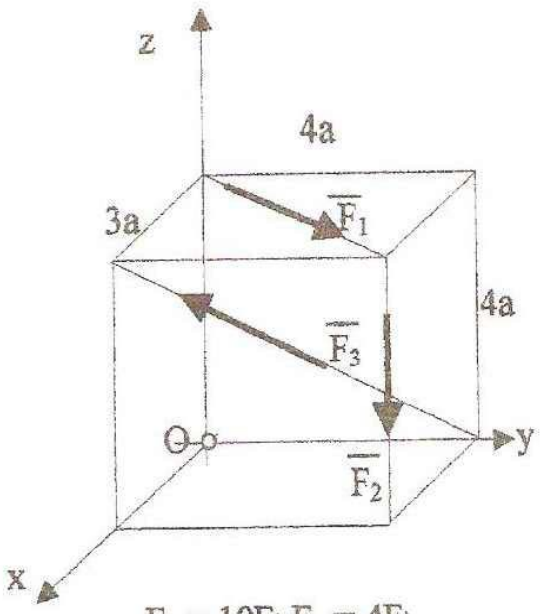
$F_1=3F;\ F_2=4\sqrt{5}\,F;$
$F_3=\sqrt{70}\,F$

Fig.I.38.



$F_1=F$; $F_2=2\sqrt{17}\,F$;
$F_3=2\sqrt{21}\,F$

Fig.I.39.



$F_1 = 10F$; $F_2 = 4F$;
$F_3 = 3\sqrt{41}\,F$

Fig.I.40.

CAPITOLUL II. CENTRE DE GREUTATE PENTRU PLACI PLANE OMOGENE

Problemele II.1. – II.40.

Să se determine poziția centrului de greutate pentru plăcile omogene reprezentate în figurile II.1. – II.40.

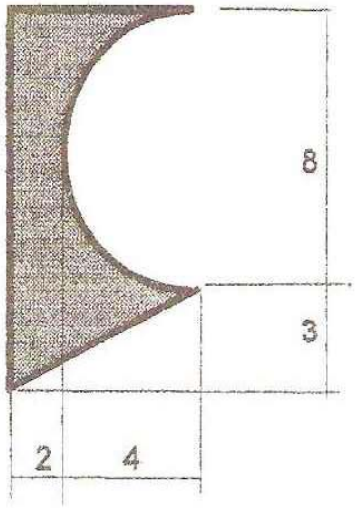


Fig.II.1.

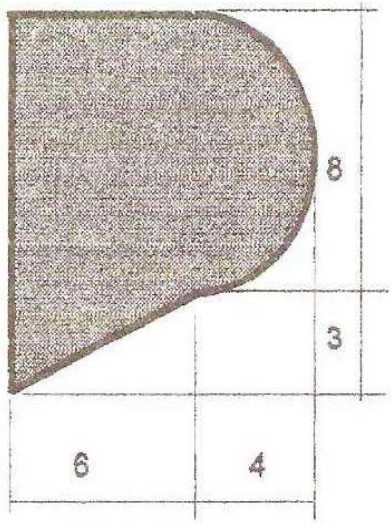


Fig. II.2.

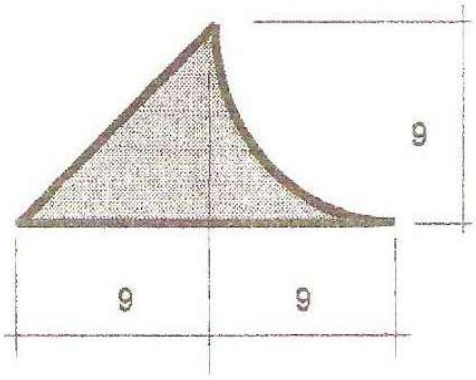


Fig. II.3.

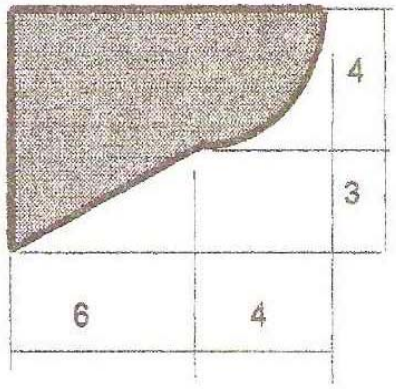


Fig. II.4.

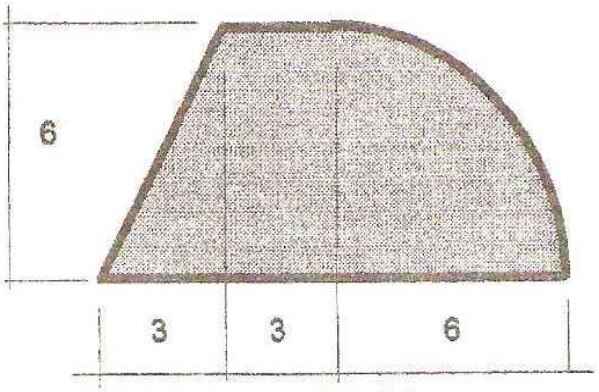


Fig. II.5.

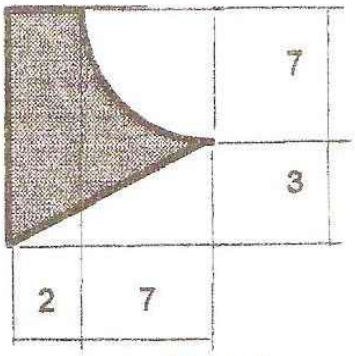


Fig. II.6.

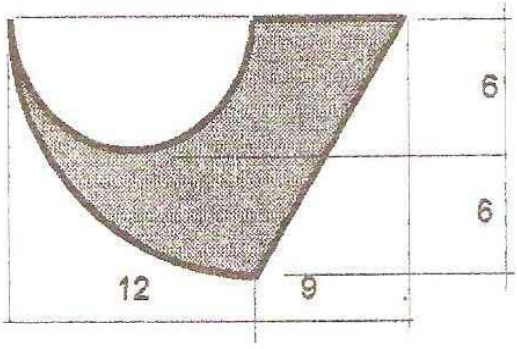


Fig.II.7.

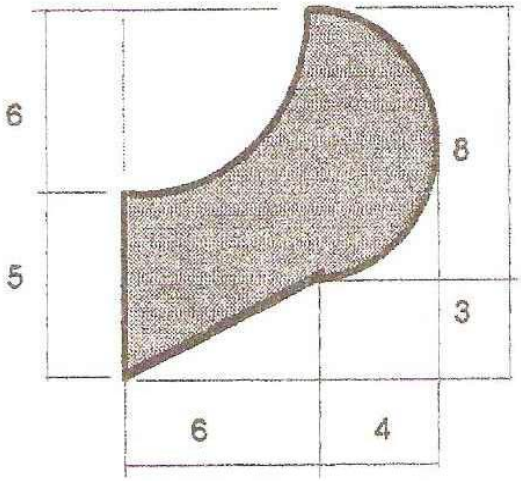


Fig.II.8.

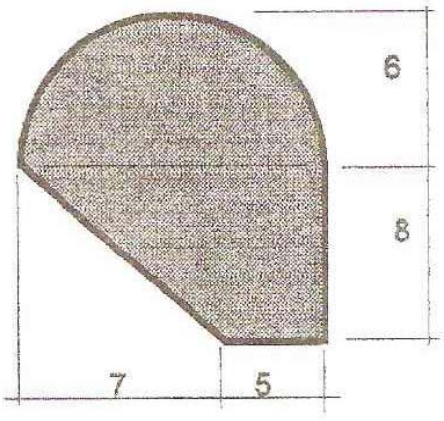


Fig.II.9.

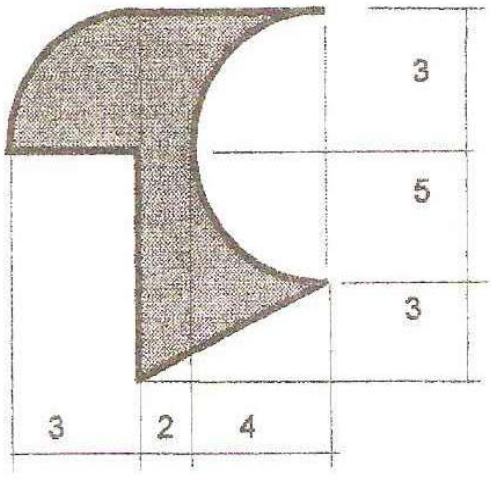


Fig.II.10.

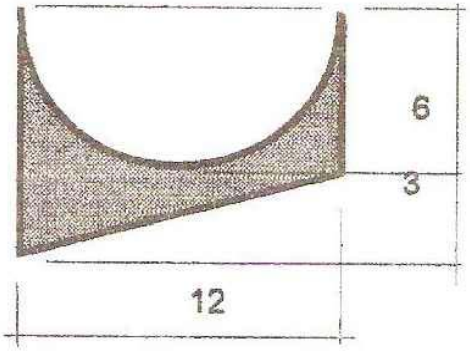


Fig.II.11.

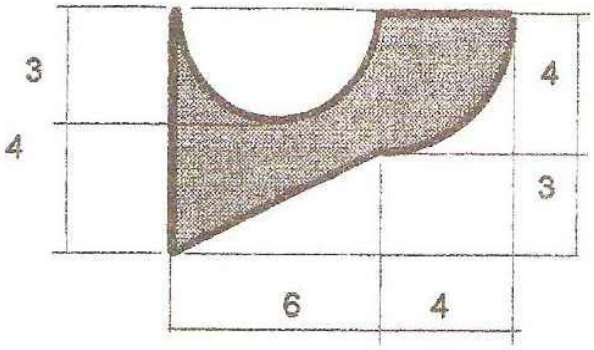


Fig.II.12.

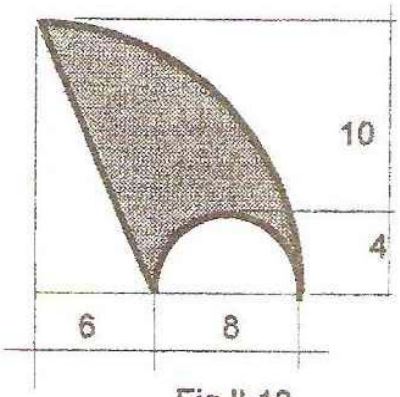


Fig.II.13.

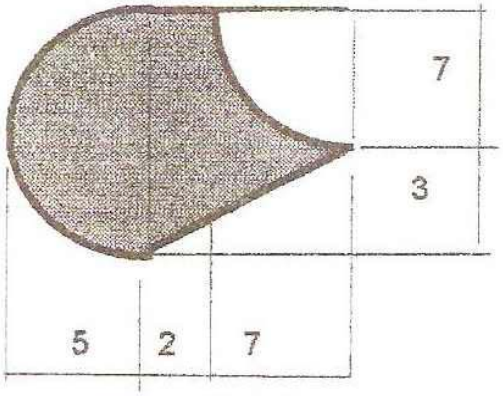


Fig.II.14.

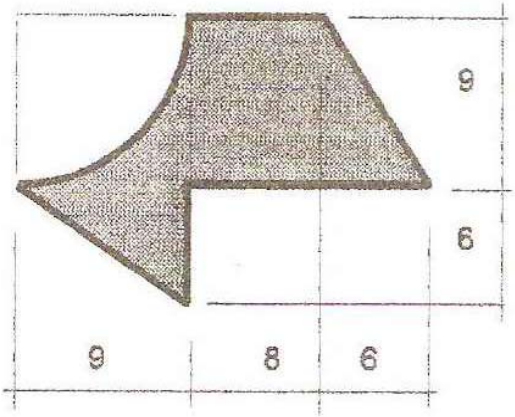


Fig.II.15.

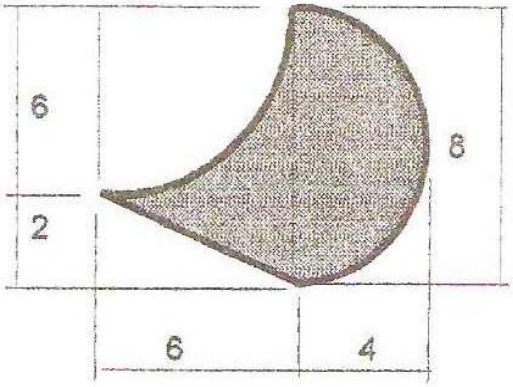


Fig.II.16.

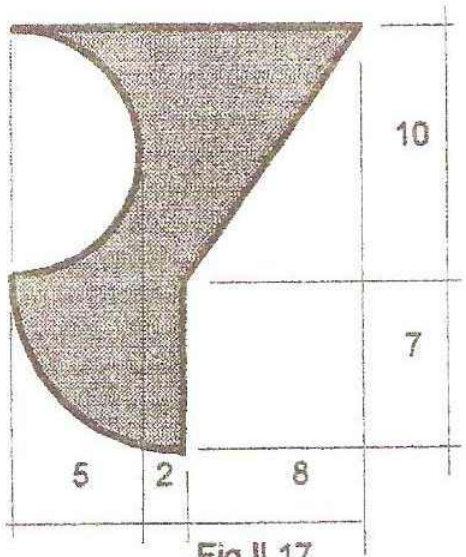
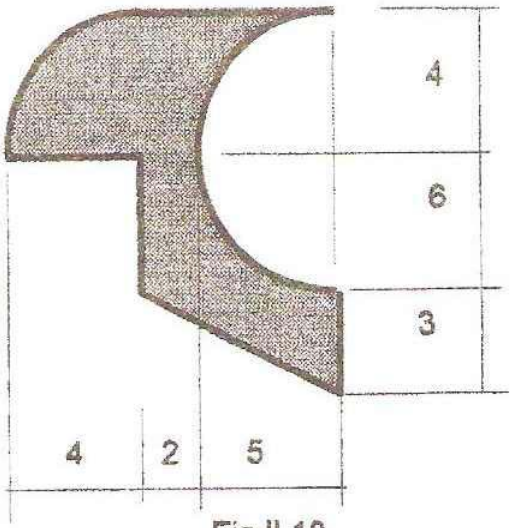


Fig.II.17.



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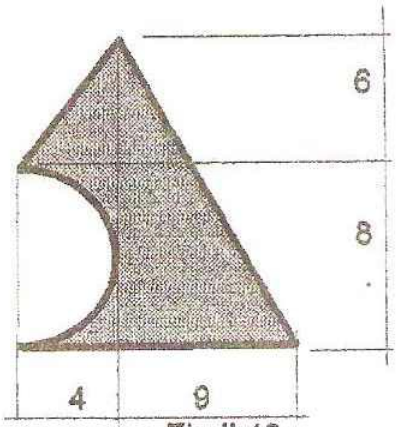


Fig.II.19.

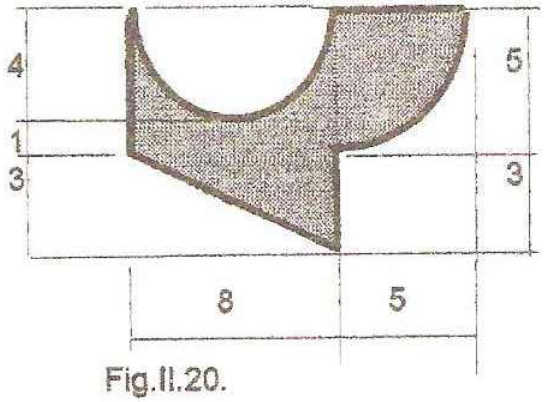


Fig.II.20.

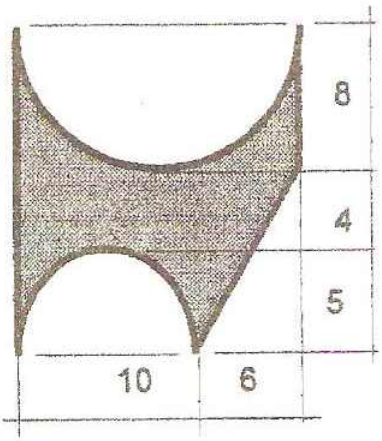


Fig.II.21.

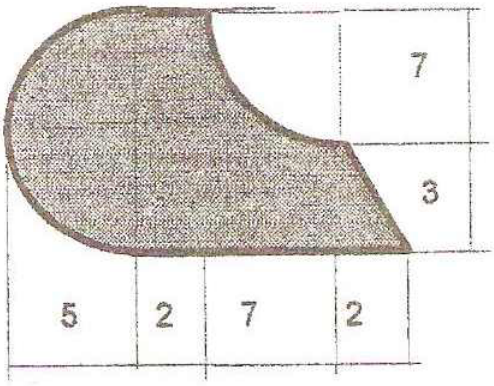


Fig.II.22.

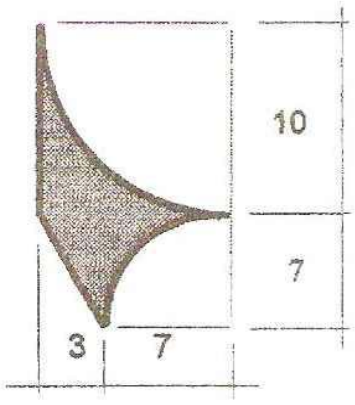


Fig.II.23.

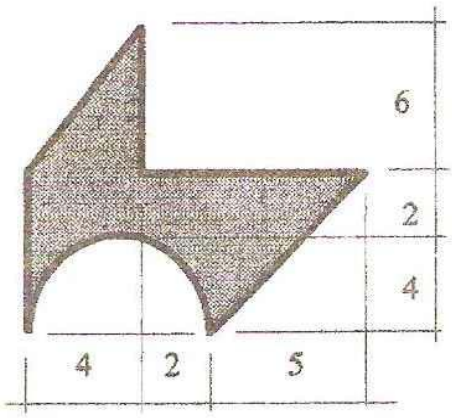


Fig.II.24.

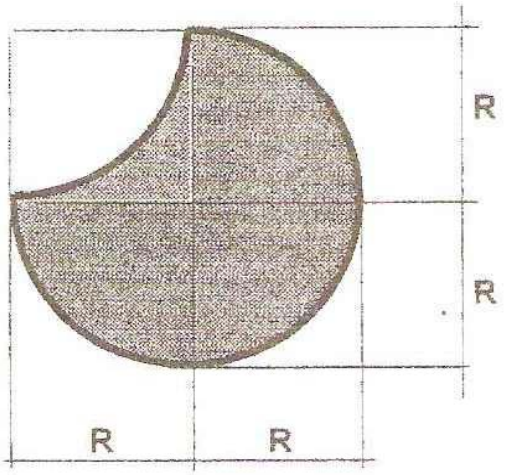


Fig.II.25.

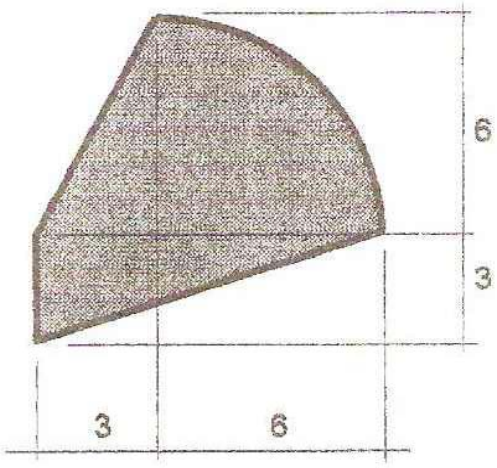


Fig.II.26

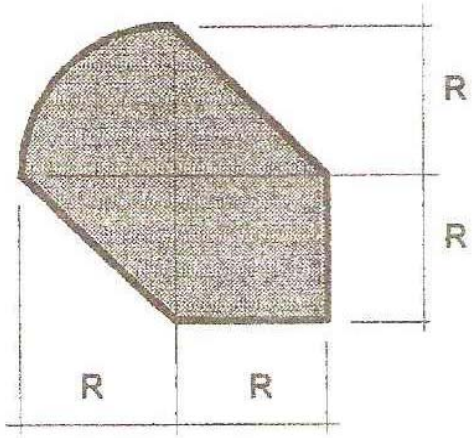


Fig.II.27.

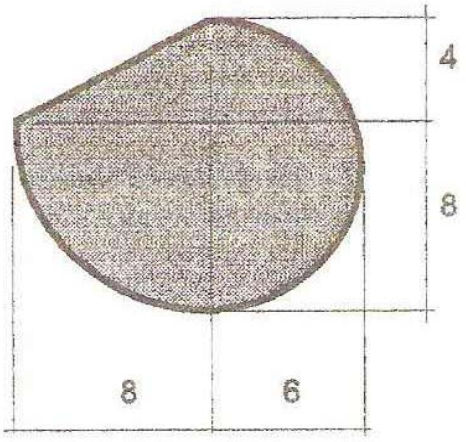


Fig.II.28

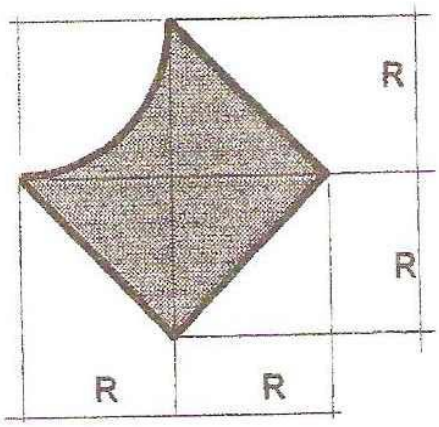


Fig.II.29.

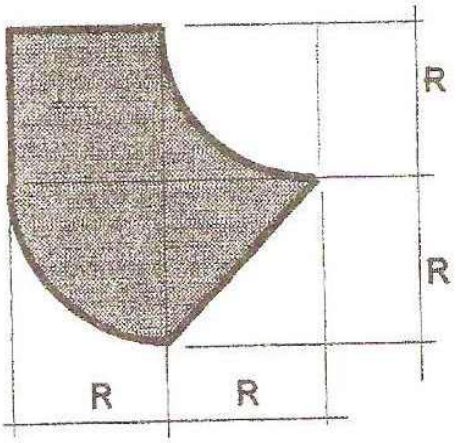


Fig.II.30.

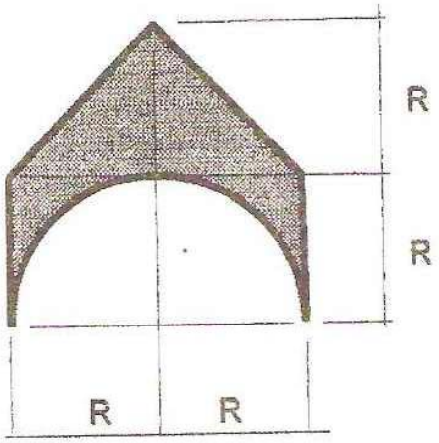


Fig.II.31.

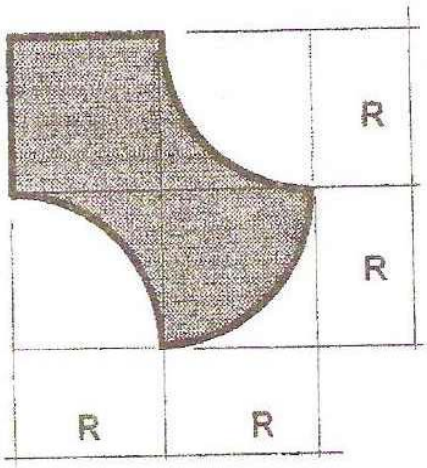


Fig.II.32.

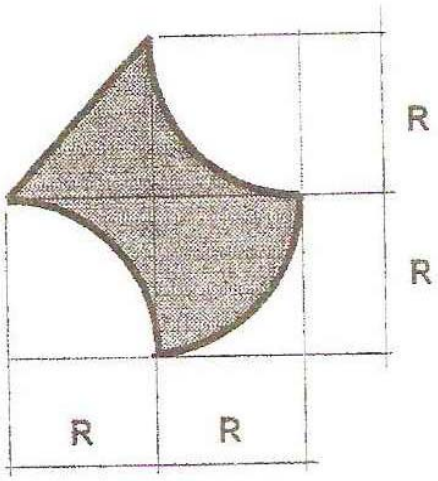


Fig.II.33.

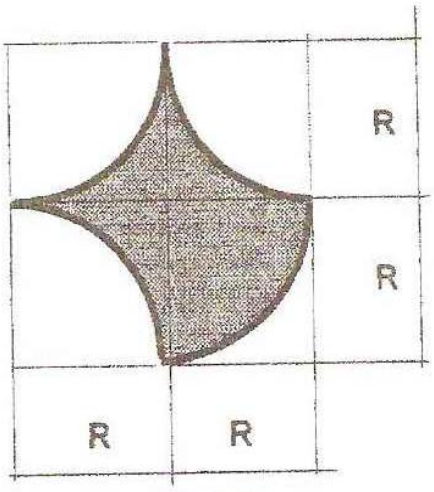


Fig.II.34.

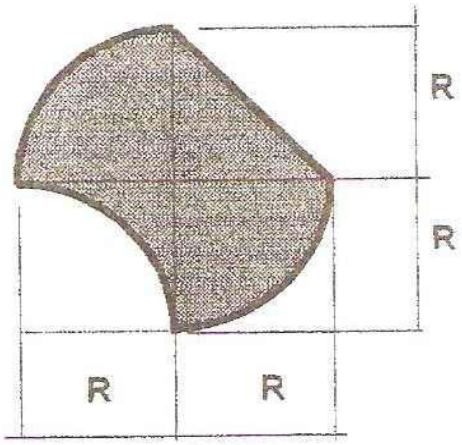


Fig.II.35.

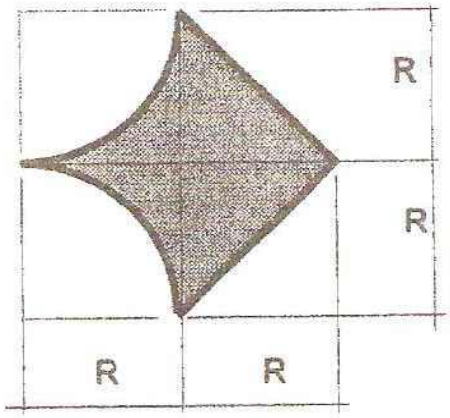


Fig.II.36.

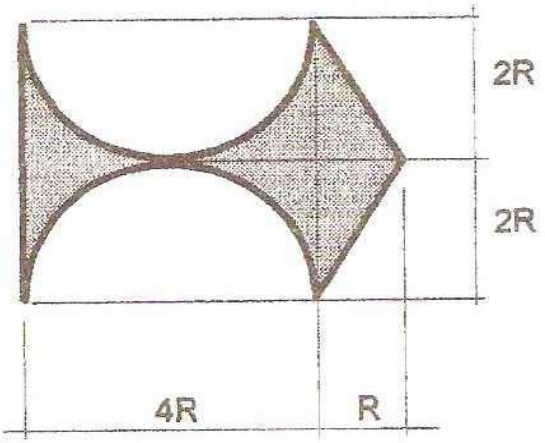


Fig.II.37.

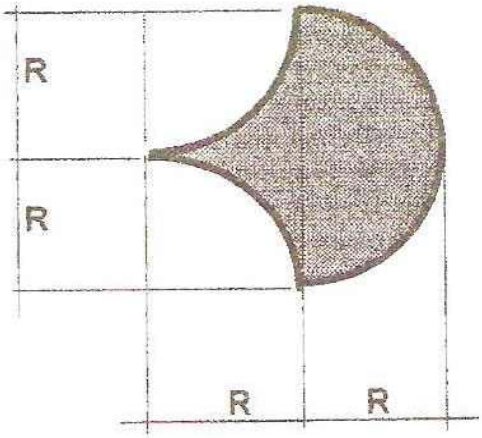


Fig.II.38.

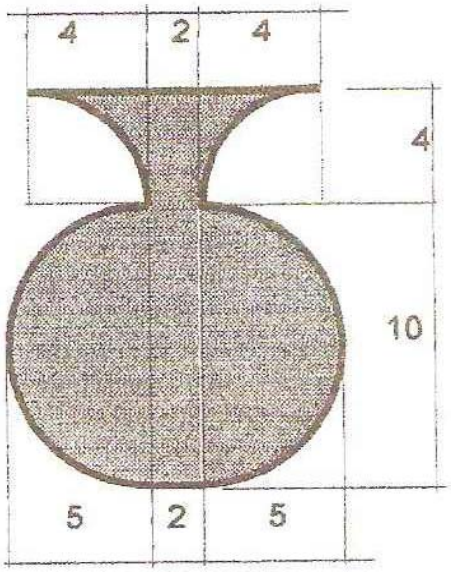


Fig.II.39.

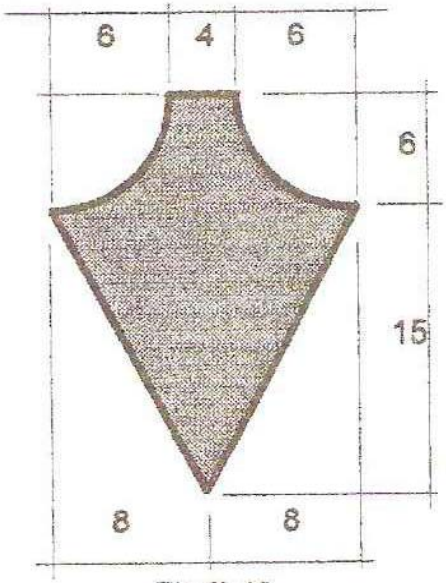


Fig.II.40.

CAPITOLUL III.CALCULUL REACTIUNILOR LA CADRE

Problemele III.1. – III.40.

Să se calculeze reacţiunile din legăturile exterioare pentru sistemele de corpuri prezentate în figurile III.1 – III.40.

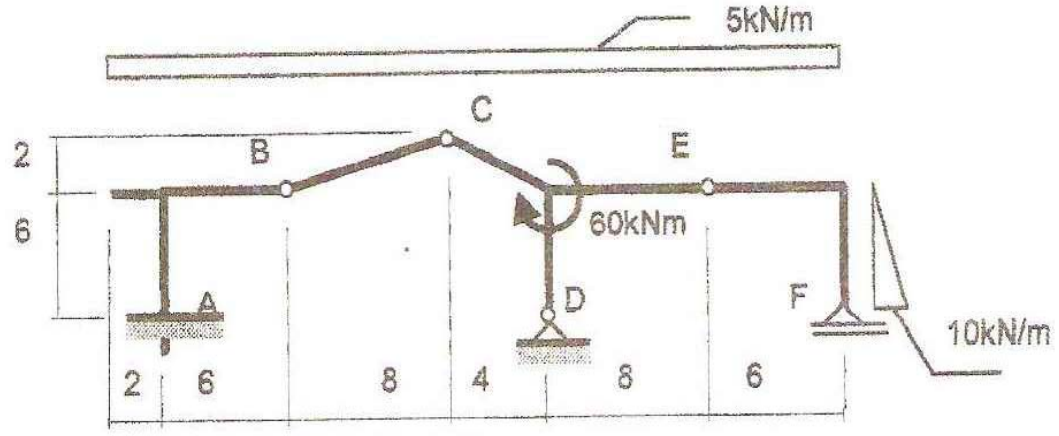


Fig.III.1.

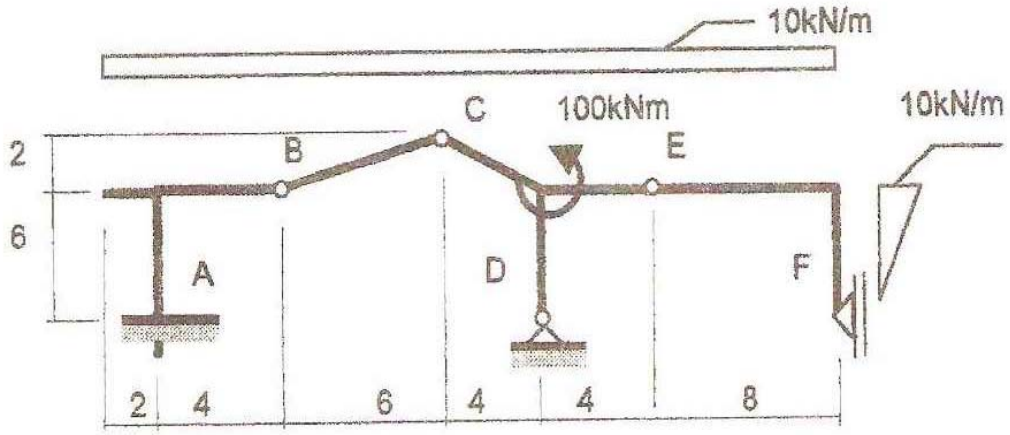


Fig.III.2.

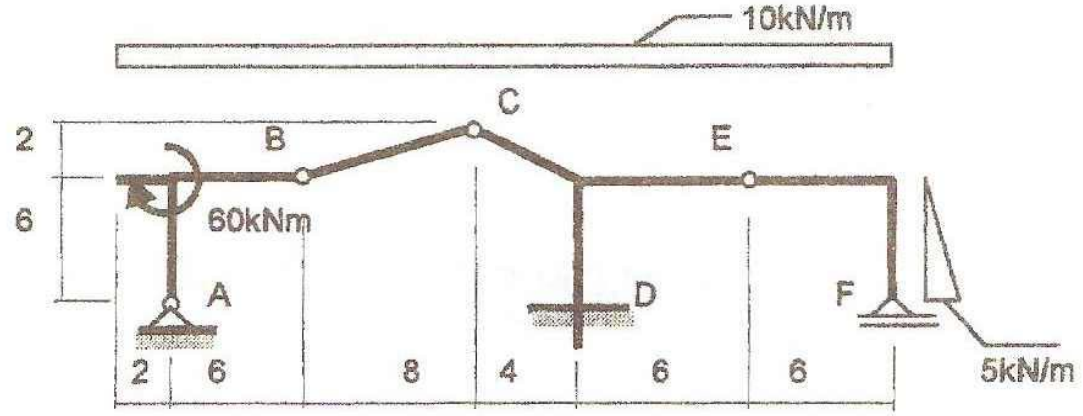


Fig.III.3.

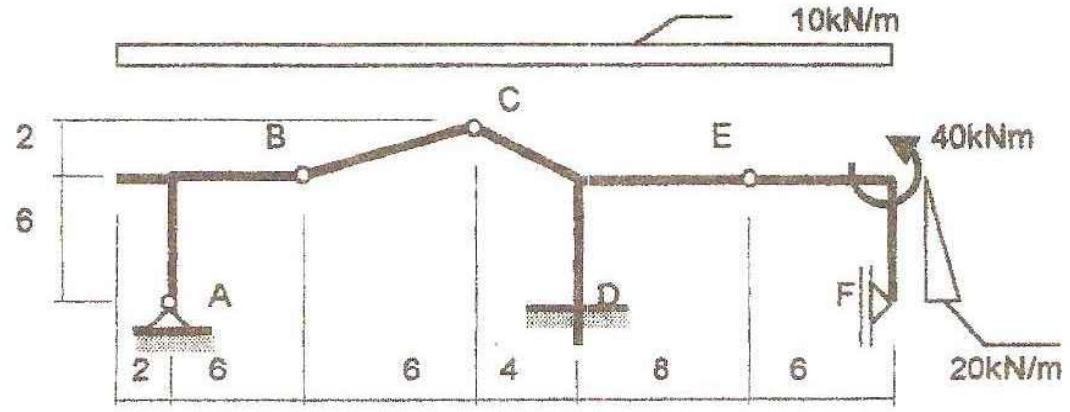


Fig.III.4.

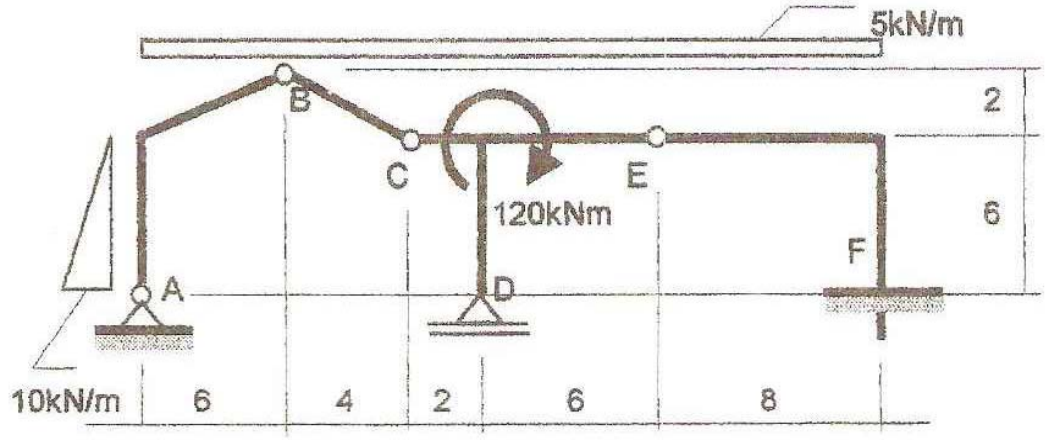


Fig.III.5.

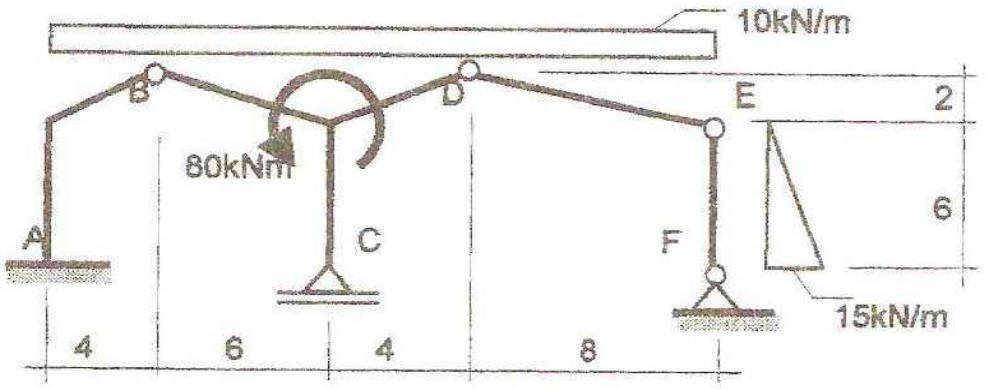
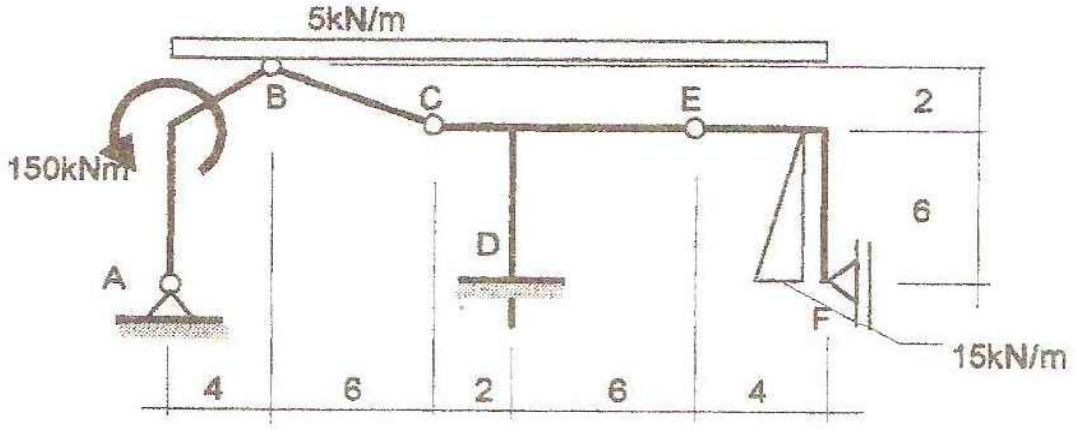


Fig.III.6.



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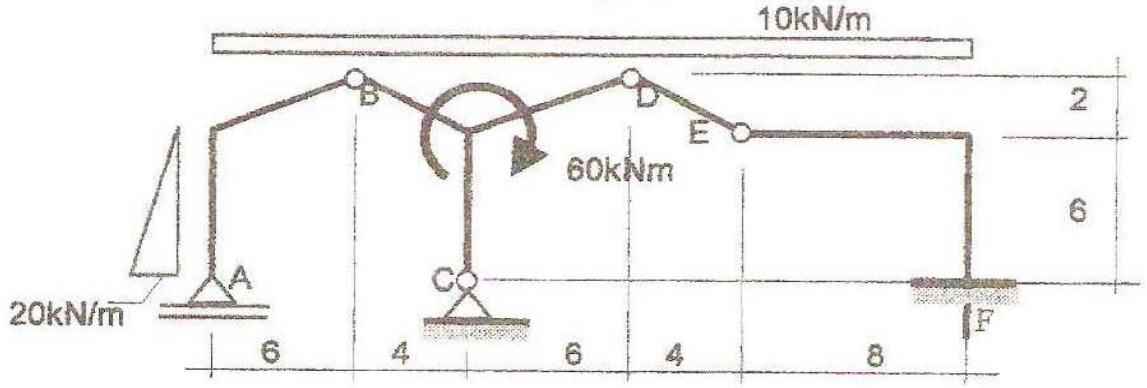

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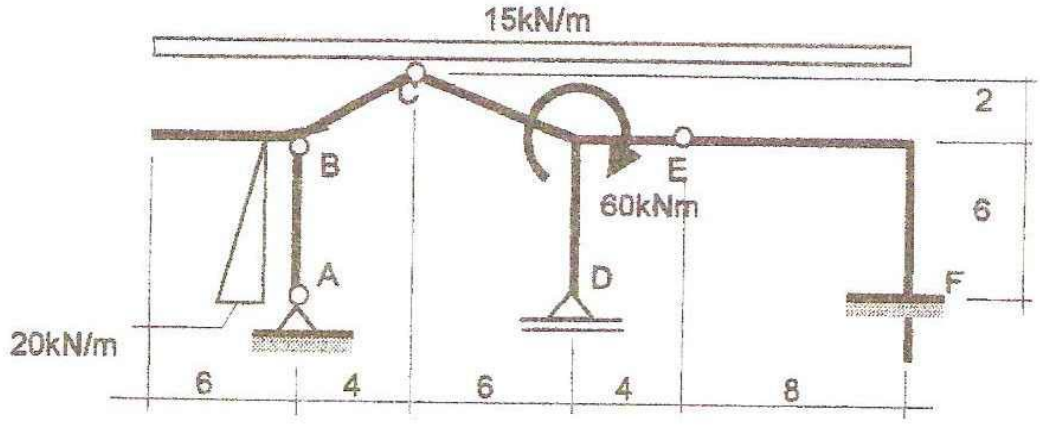

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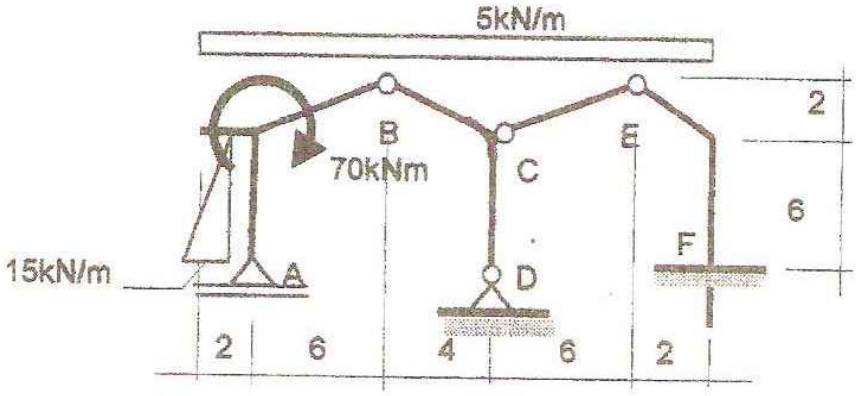

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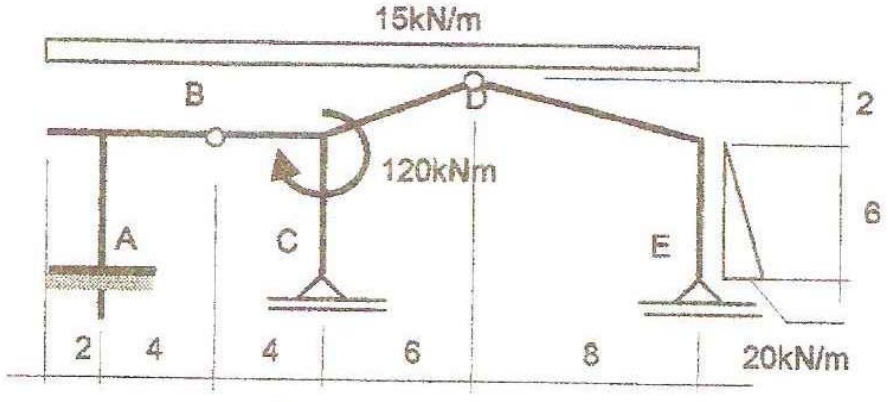

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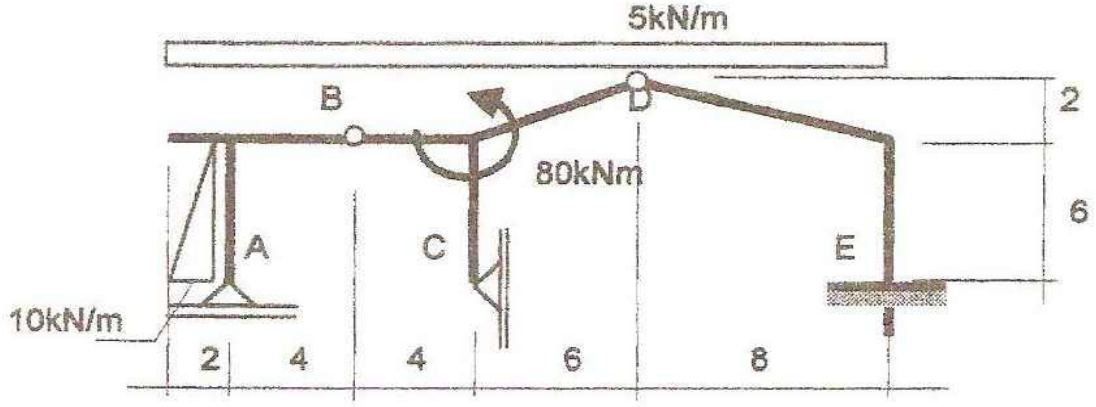

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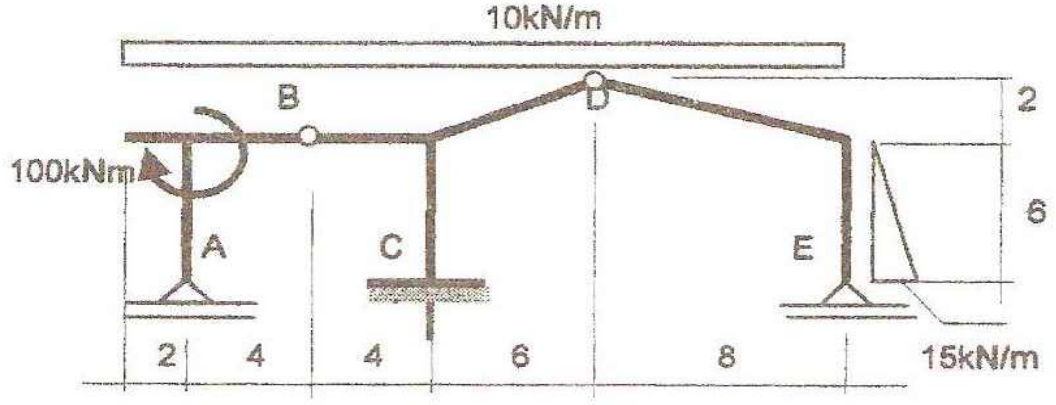

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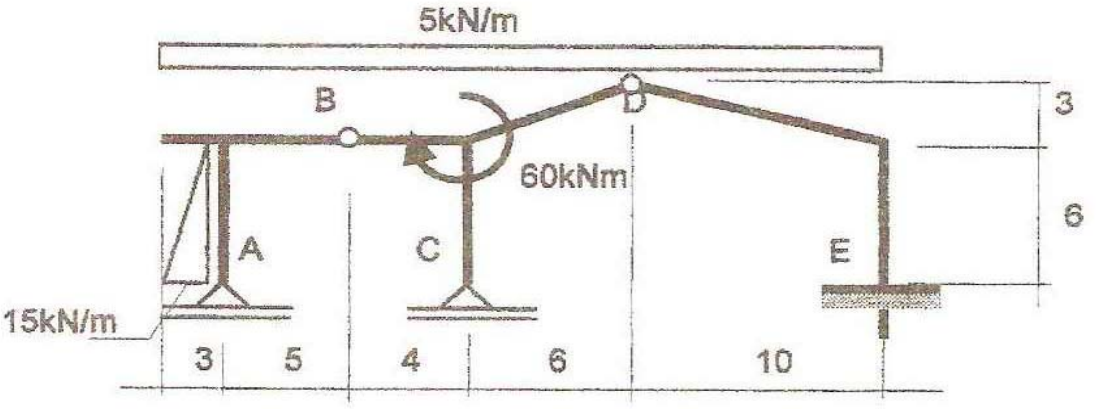

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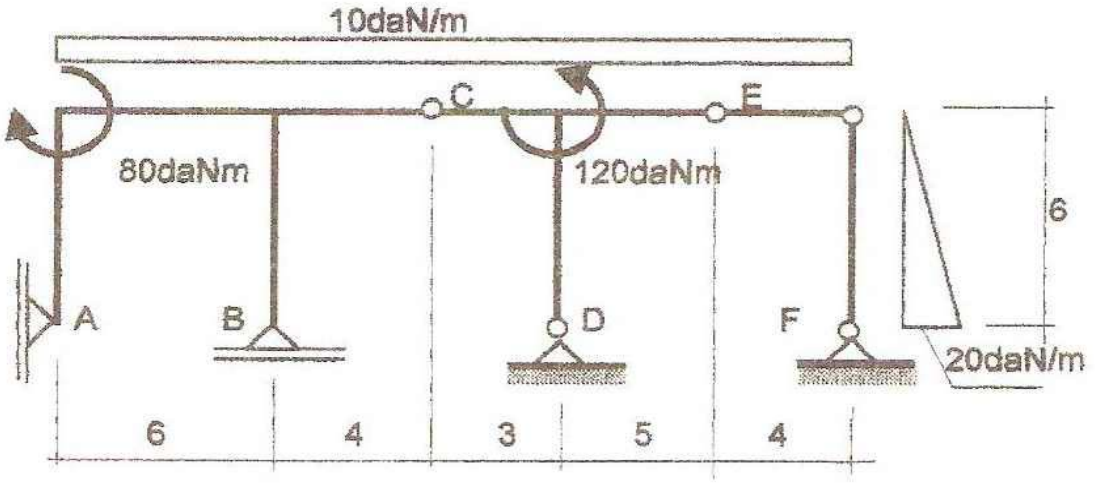

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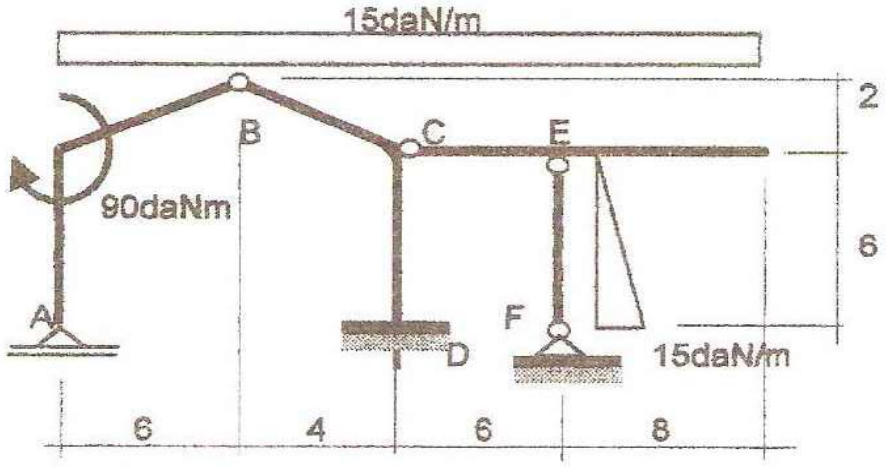

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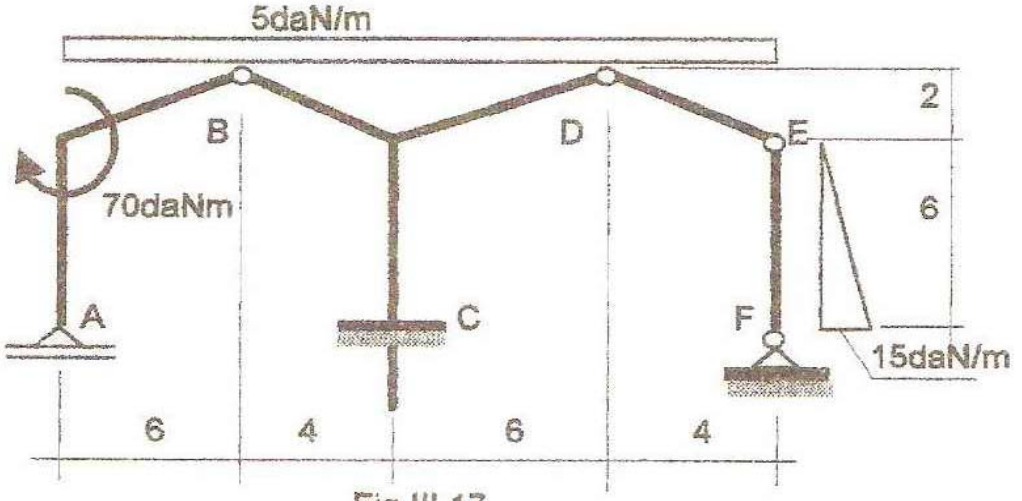

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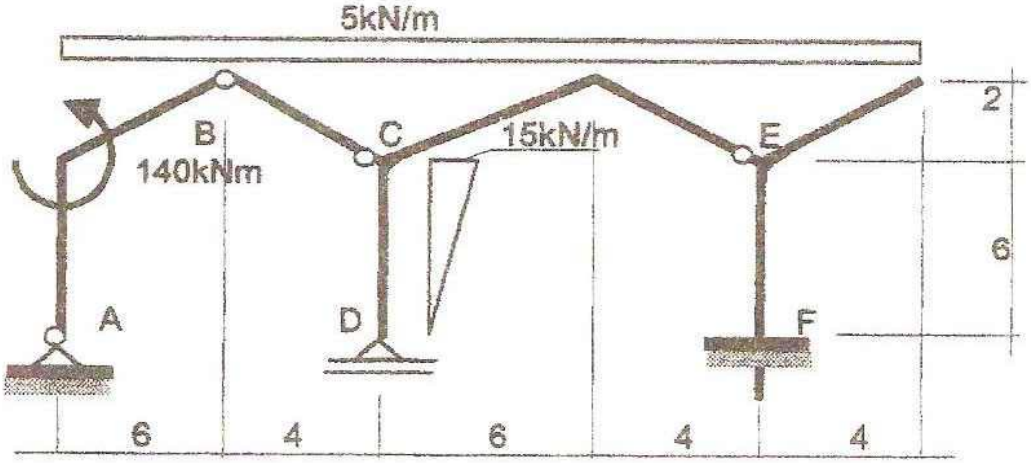

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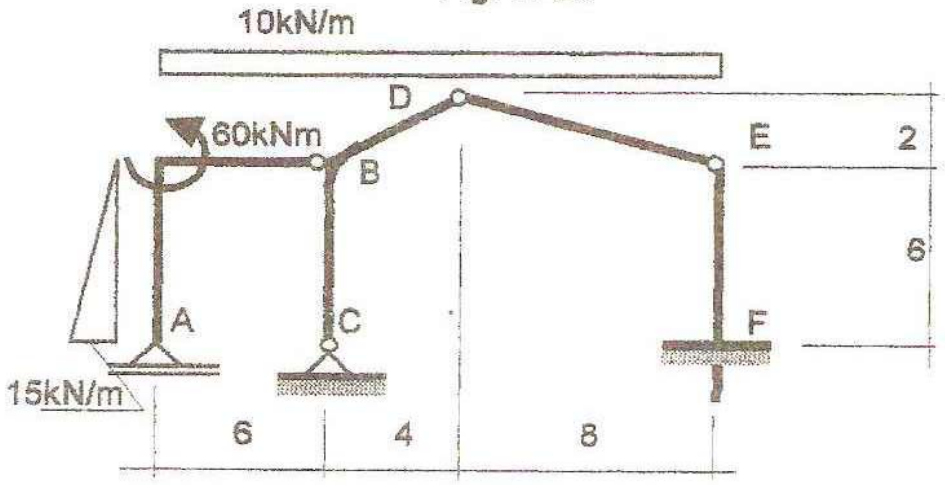

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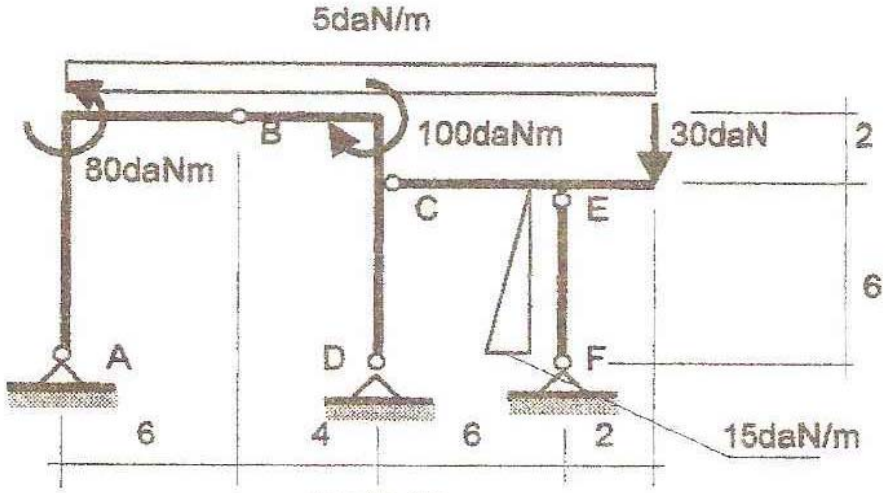

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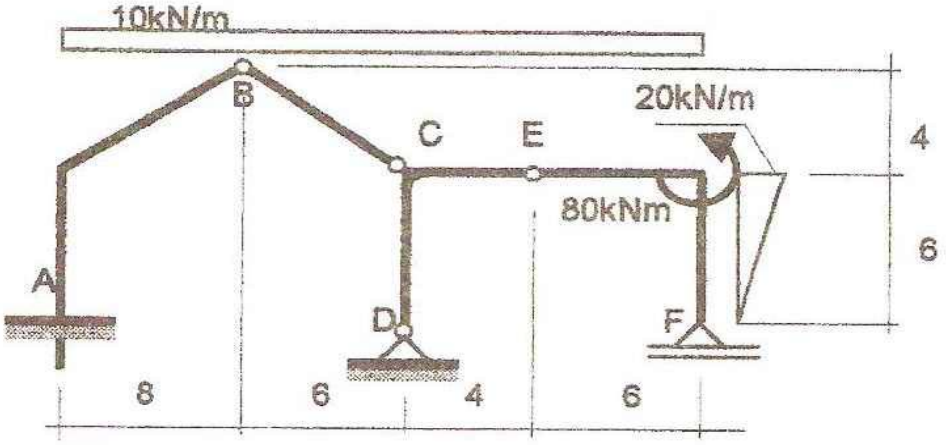

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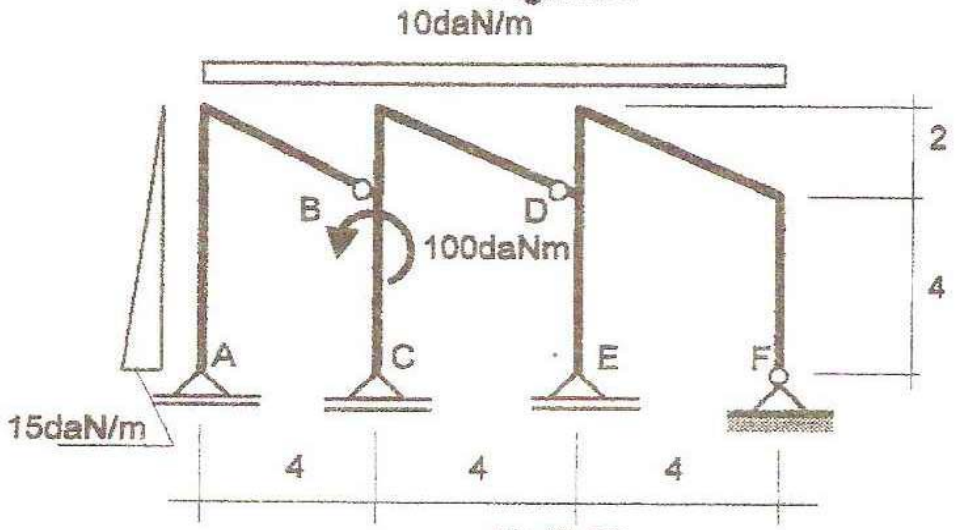

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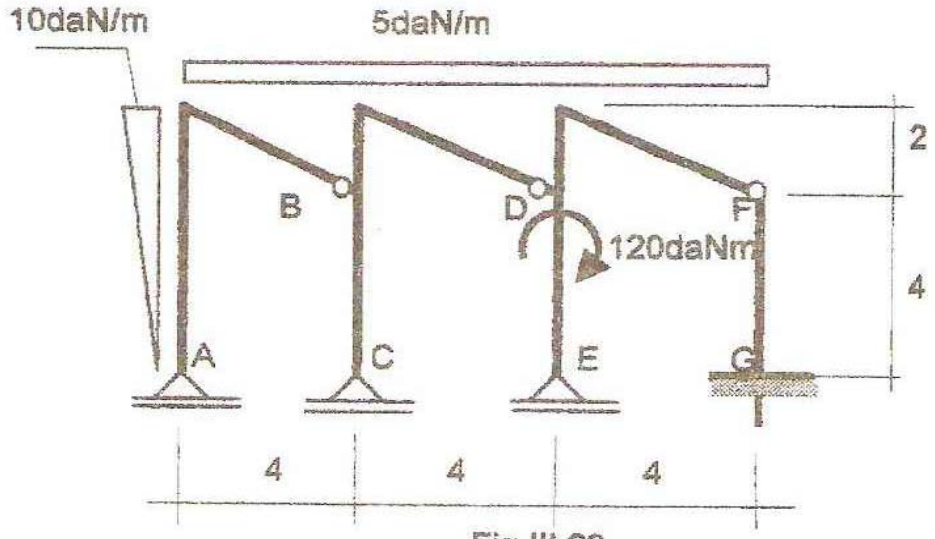

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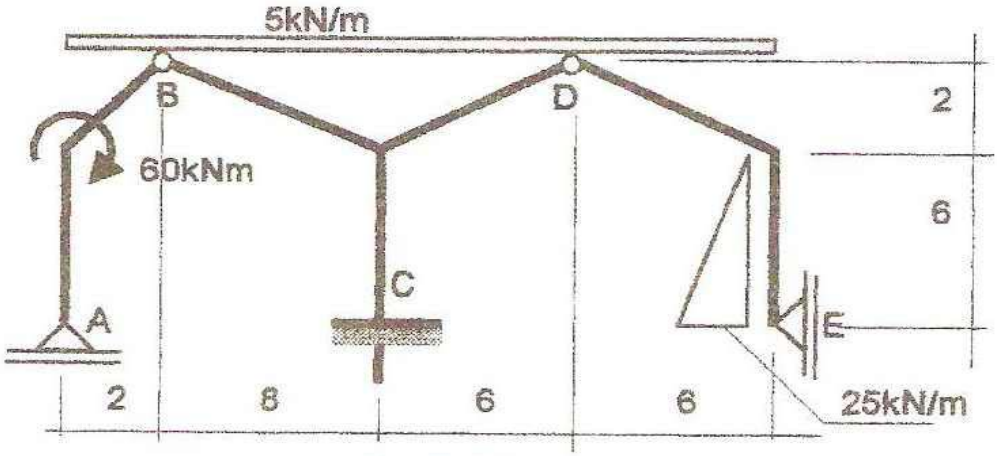

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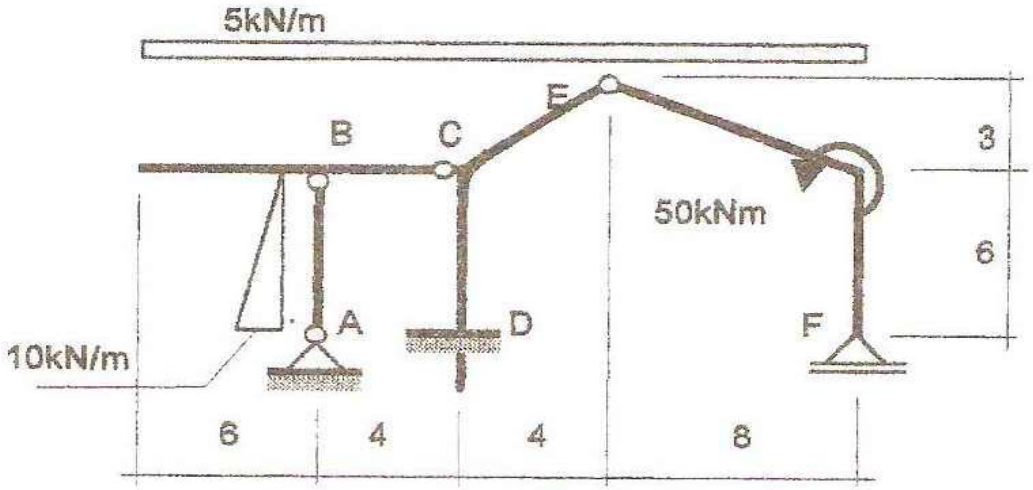

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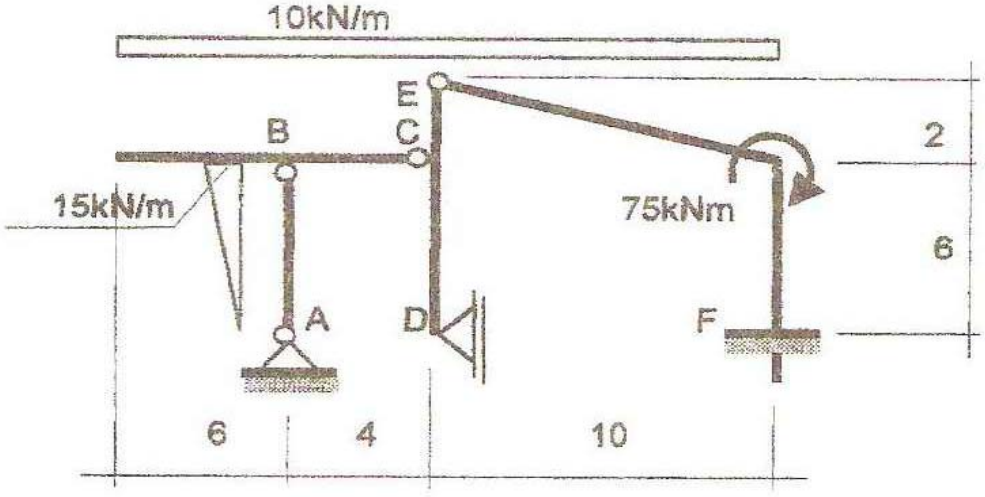

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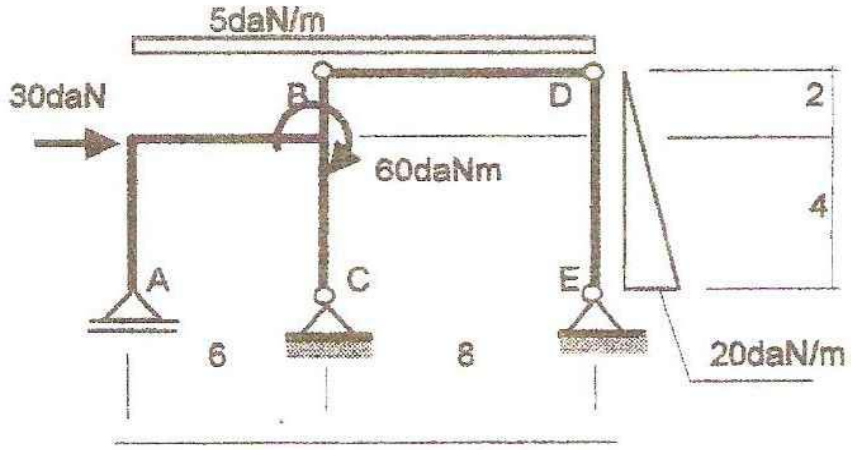

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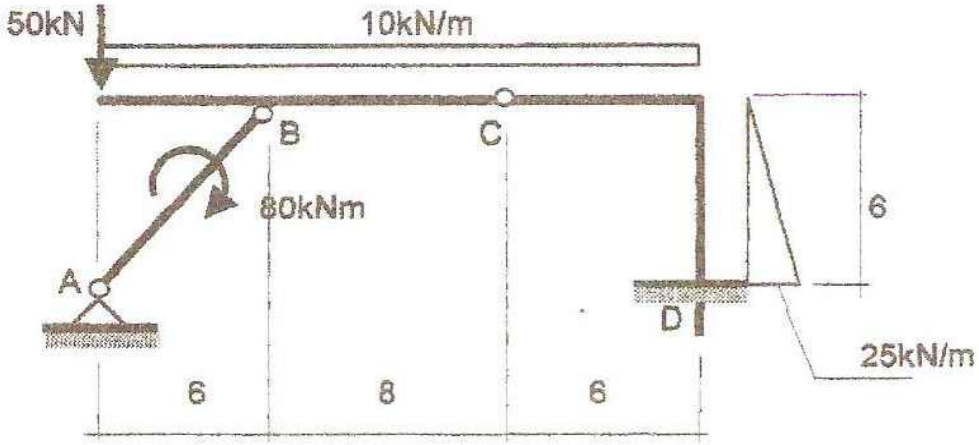

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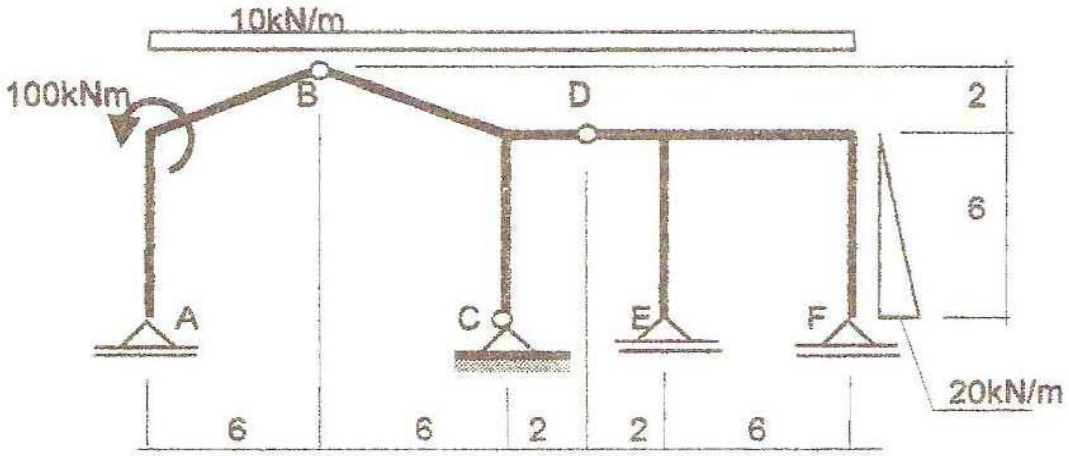

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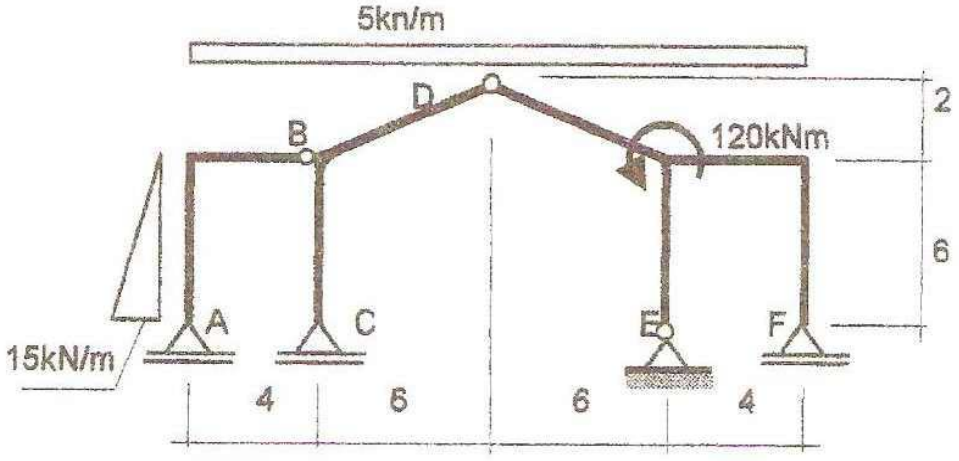

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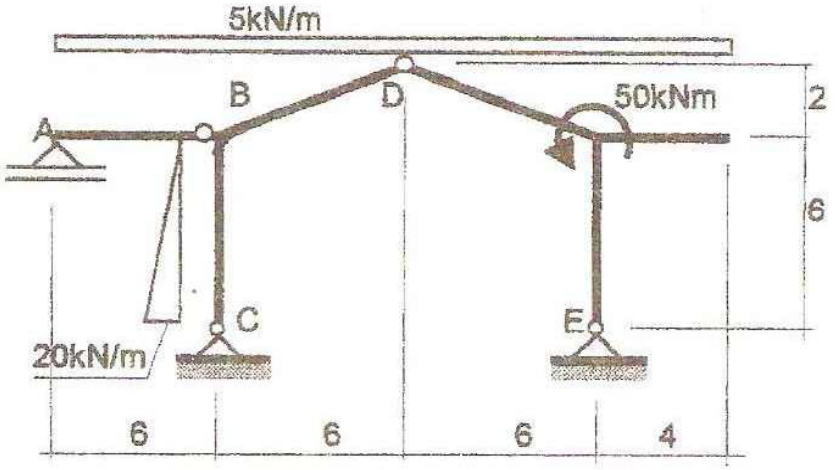

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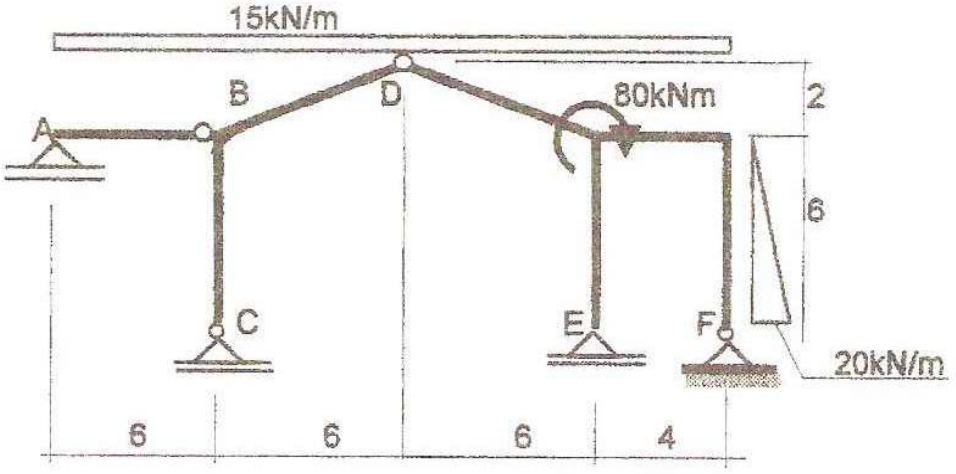

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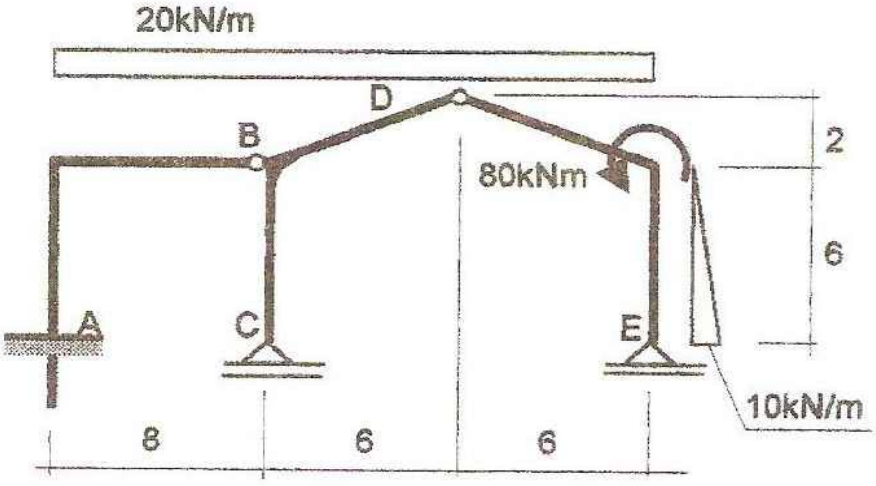

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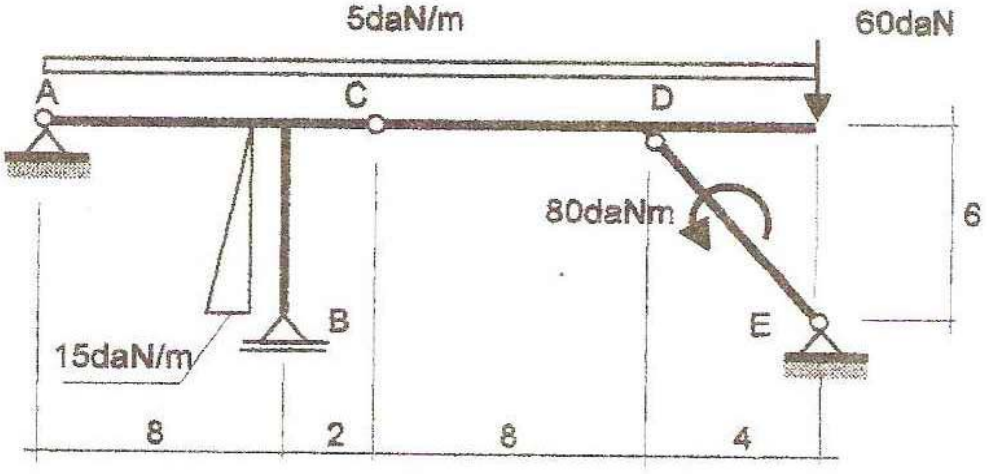

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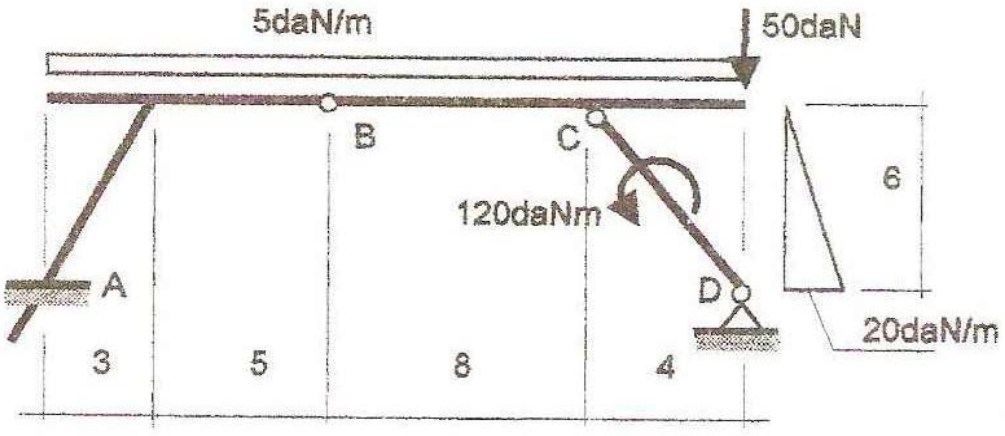

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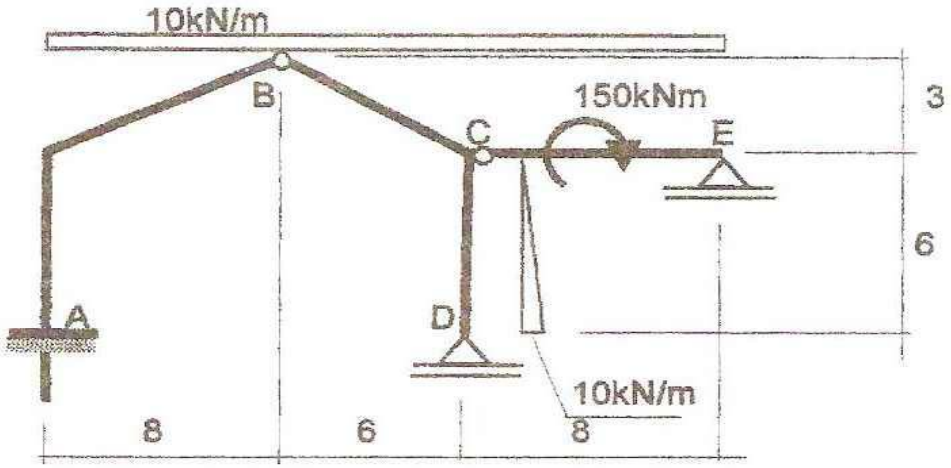

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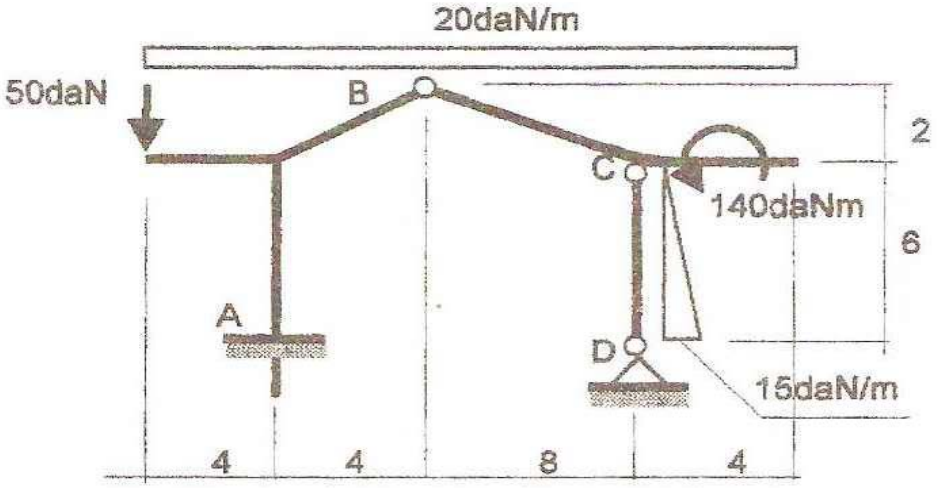

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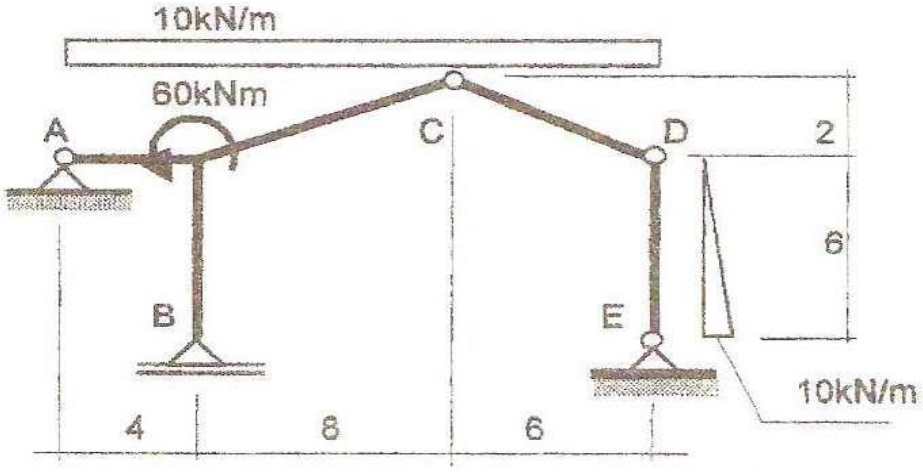

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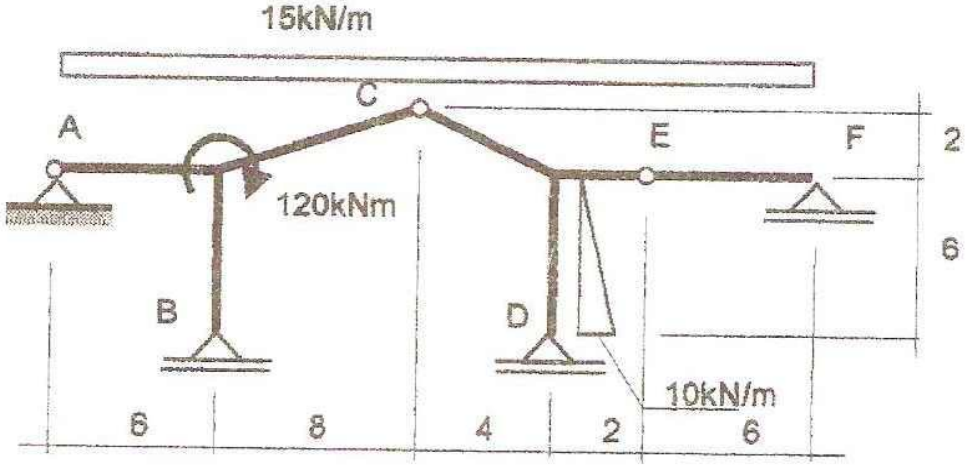

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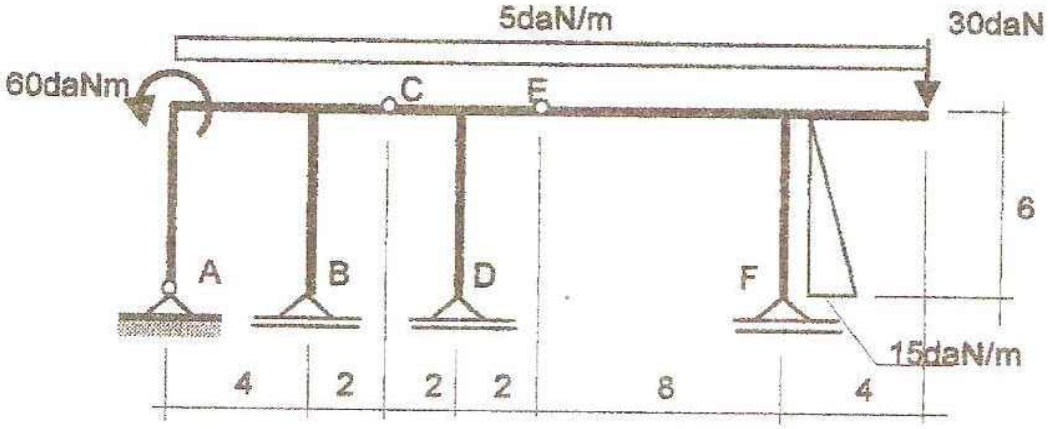


Fig.III.40.

CAPITOLUL IV. CALCULUL EFORTURILOR IN BARELE GRINZILOR CU ZABRELE SIMPLE

Problemele IV.1. – IV.40.

Folosind metoda izolării nodurilor să se calculeze eforturile în primele şase bare însemnate şi apoi folosind metoda secţiunilor să se calculeze următoarele trei eforturi din cele trei bare însemnate ale unui panou de mijloc ale grinzilor cu zăbrele simple reprezentate în figurile: IV.1. – IV.40.

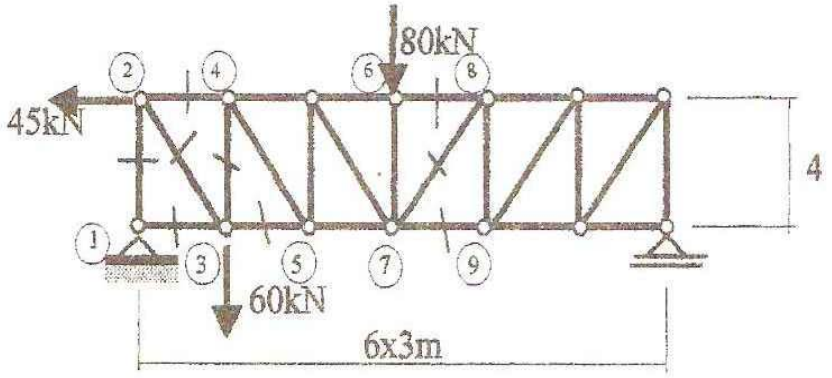


Fig. IV.1.

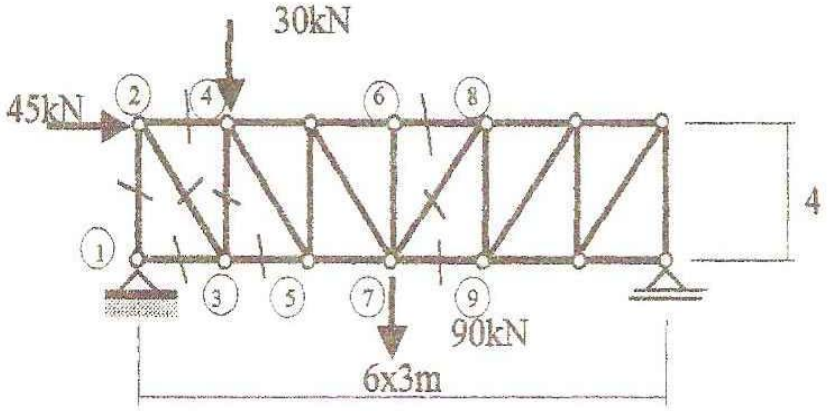


Fig. IV.2.

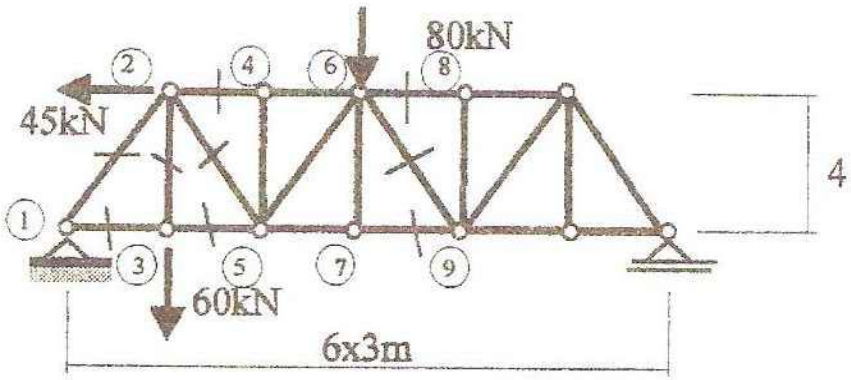


Fig. IV.3.

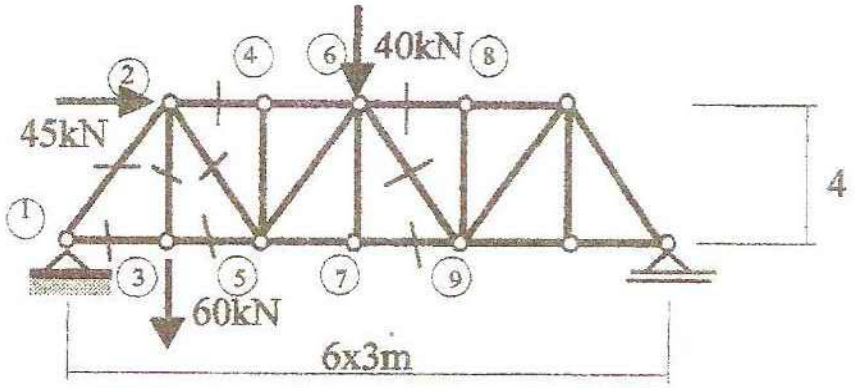


Fig. 1IV.4

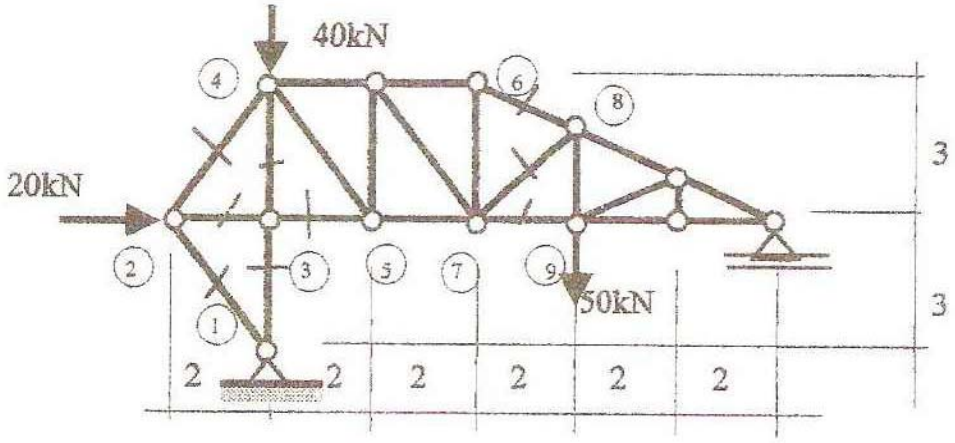
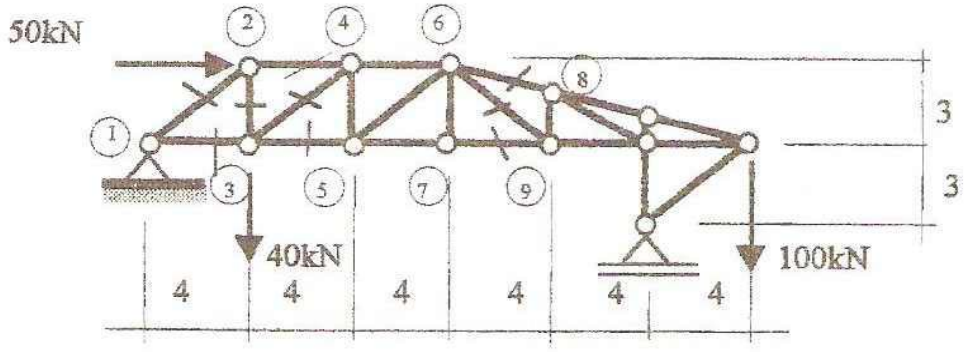


Fig. IV.5.



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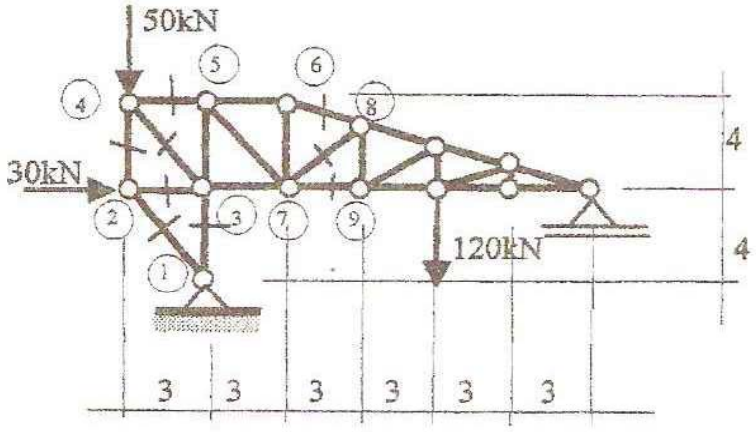

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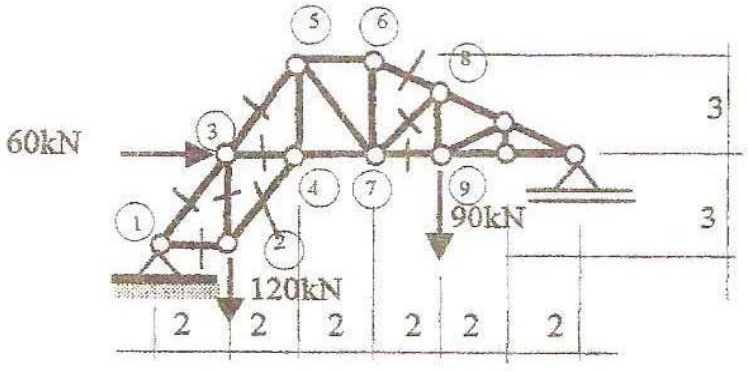
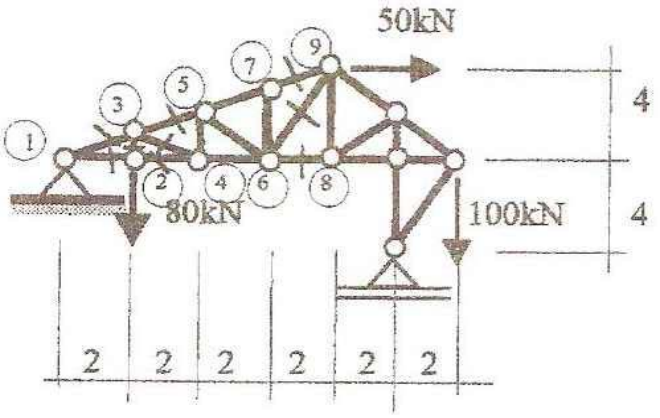


Fig. IV.8.



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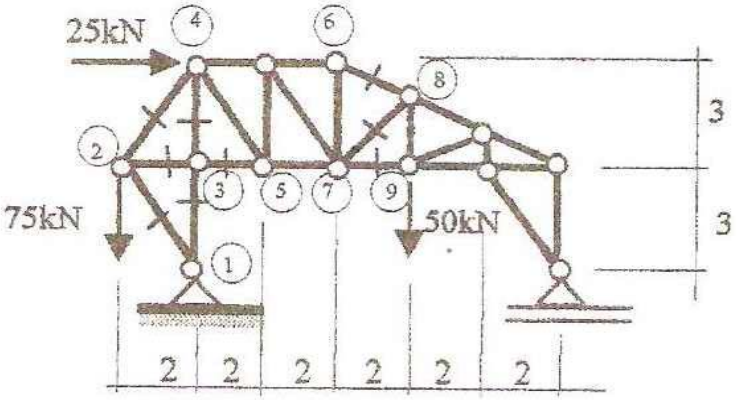

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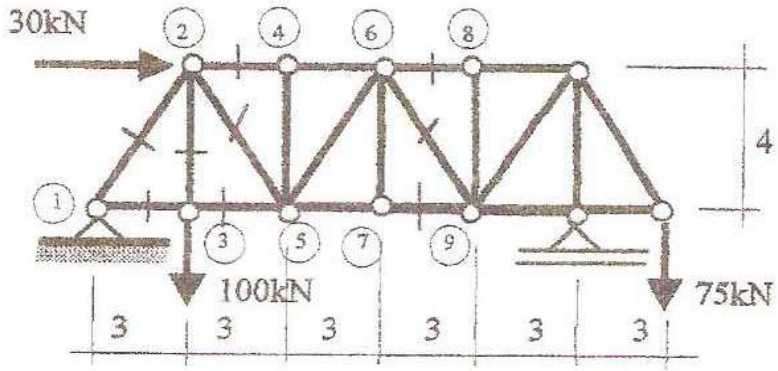

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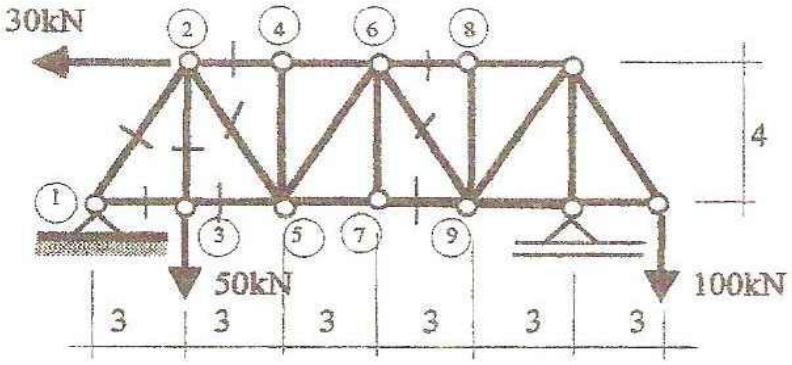

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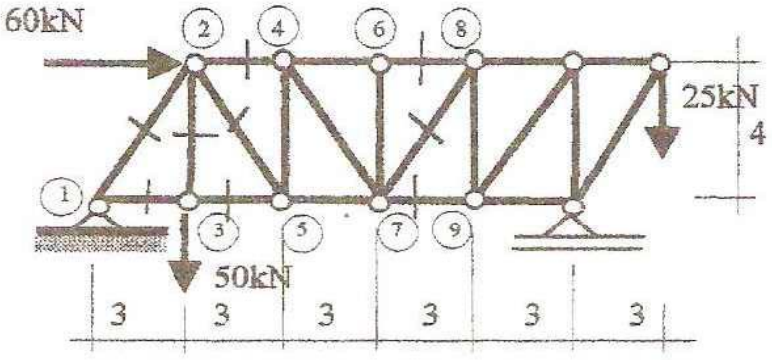

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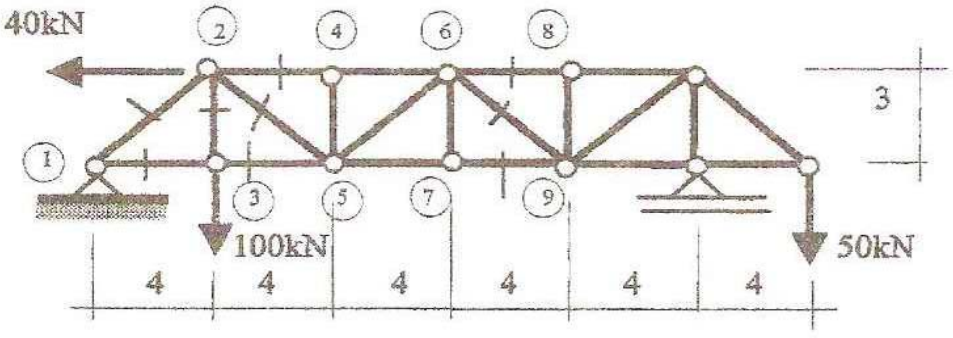

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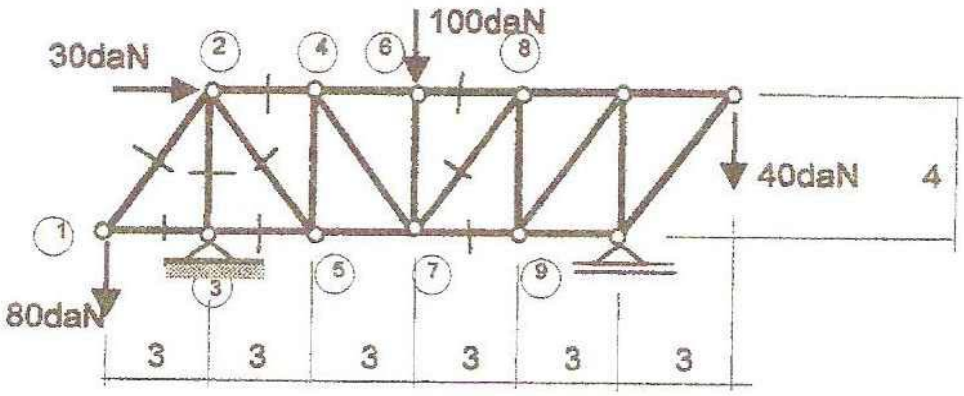

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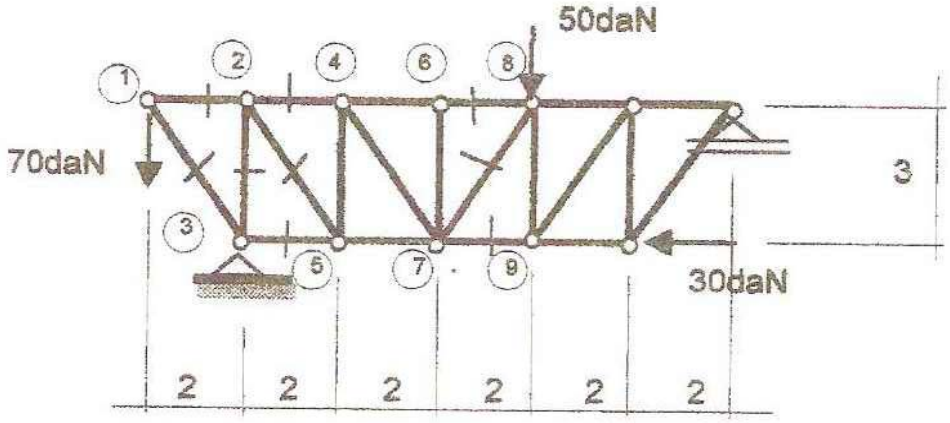

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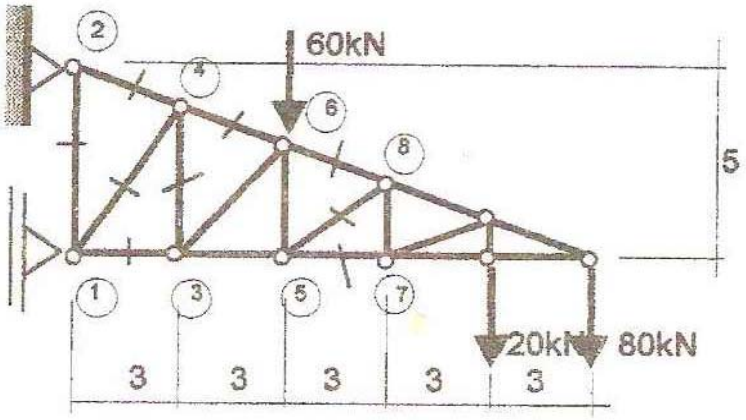

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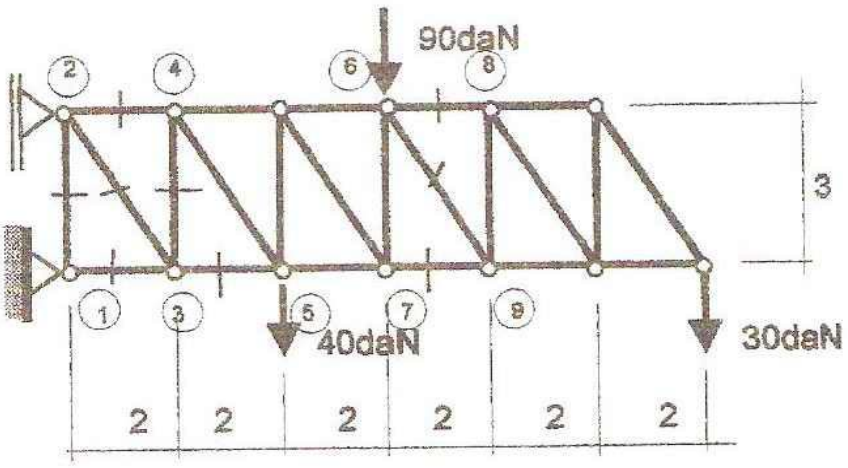

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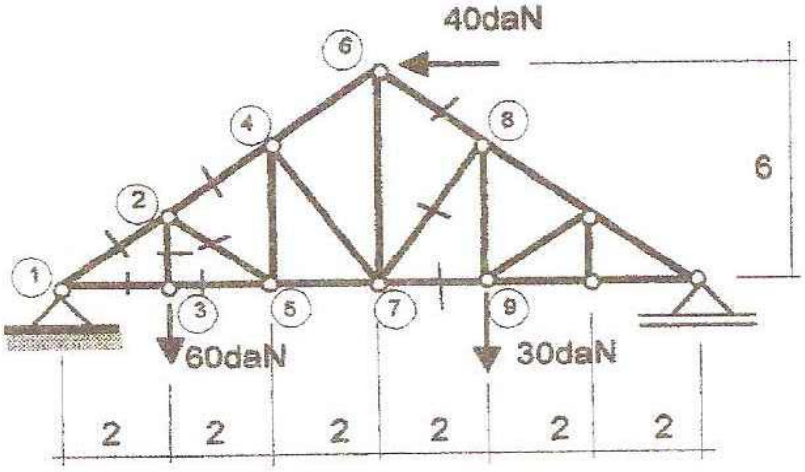
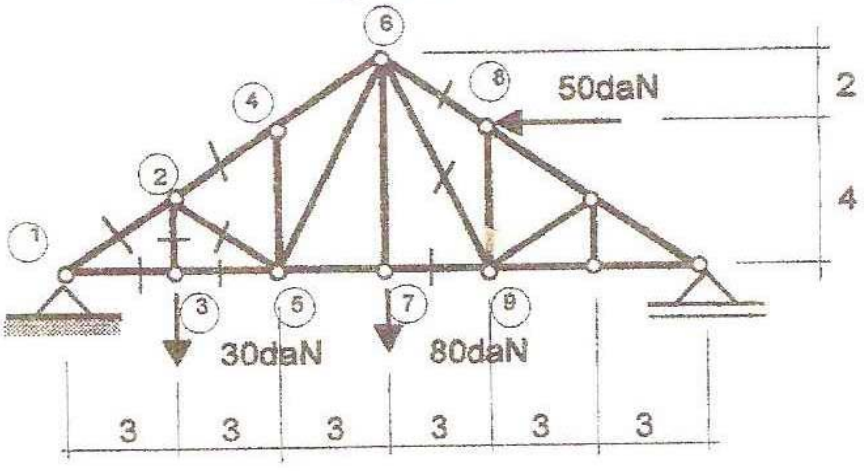


Fig.IV.19



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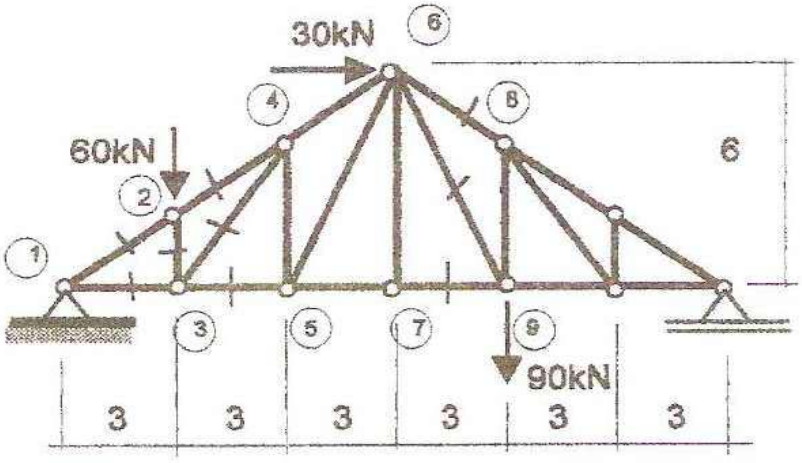

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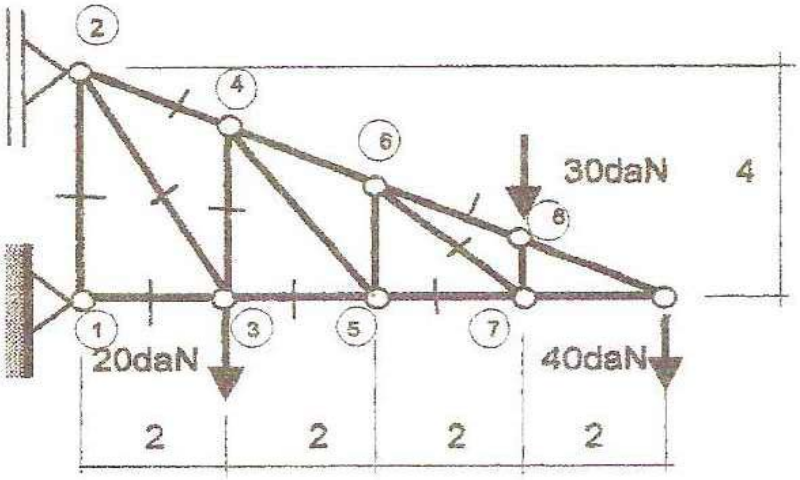

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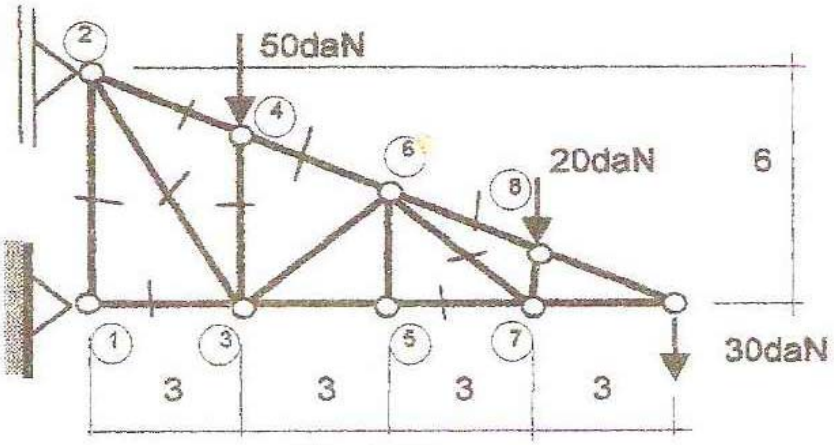

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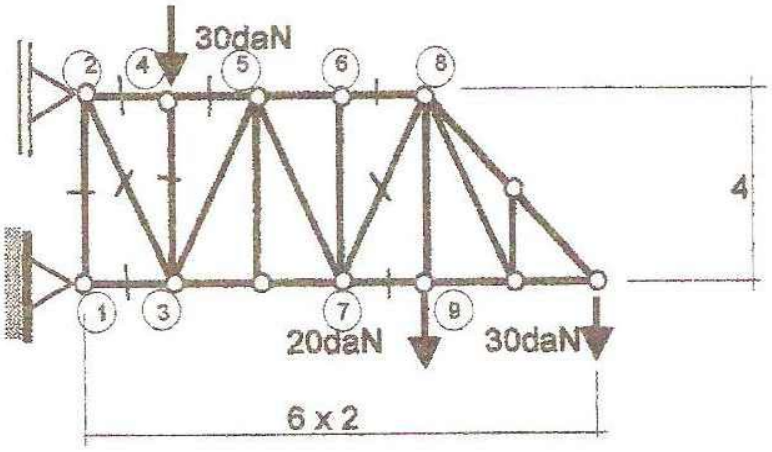

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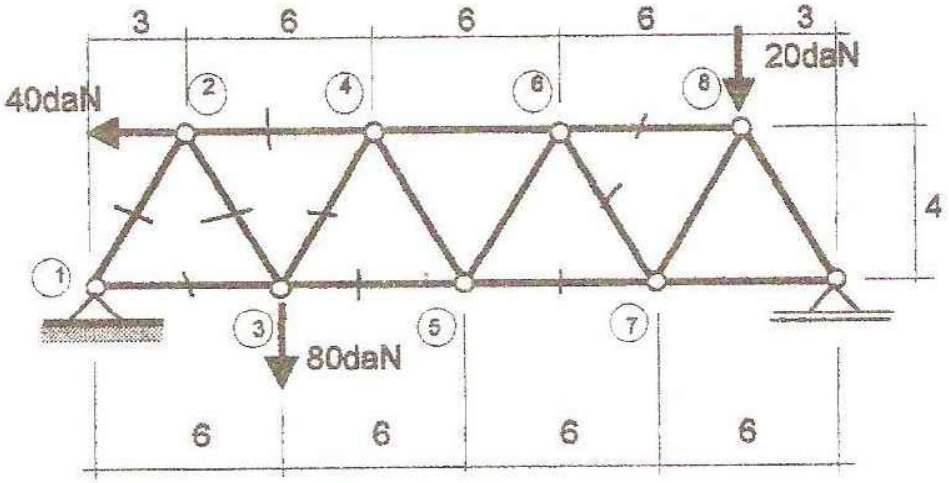

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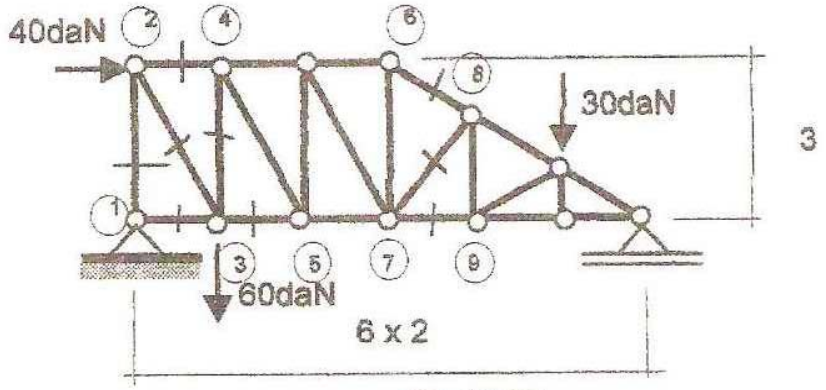

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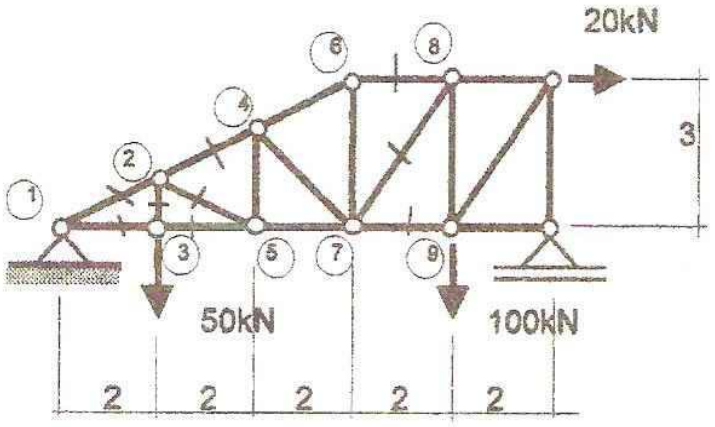

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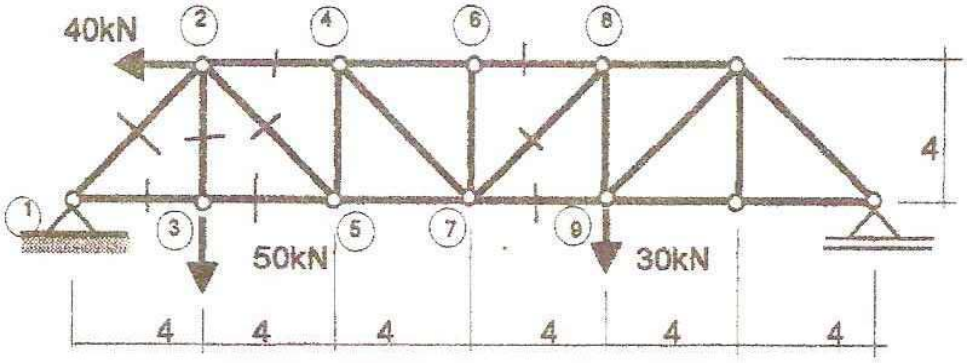

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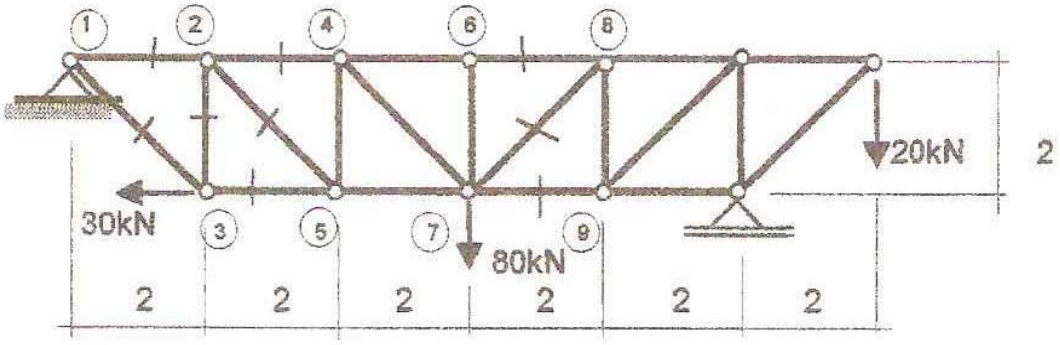

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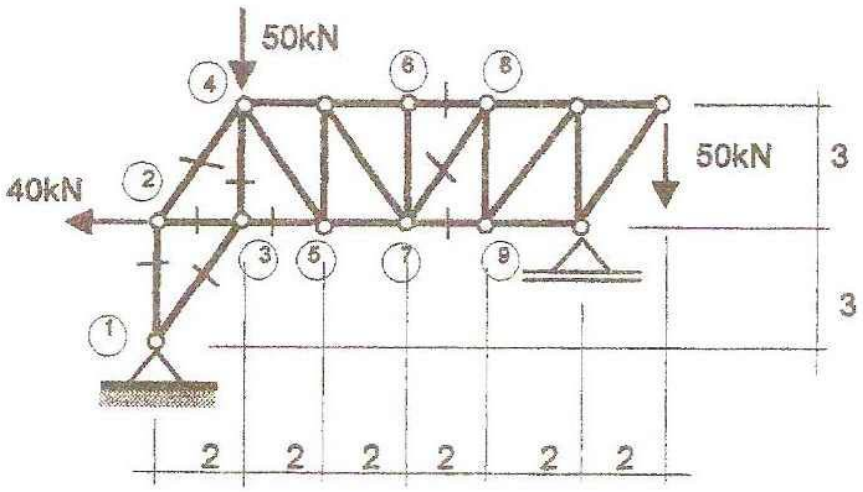

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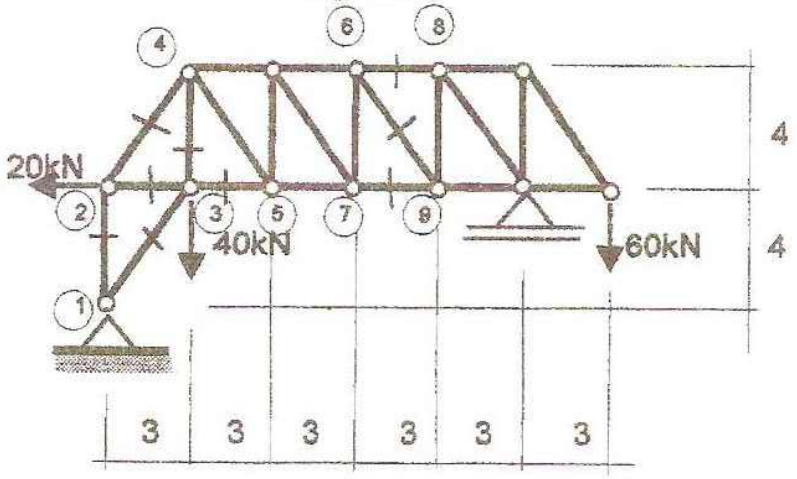

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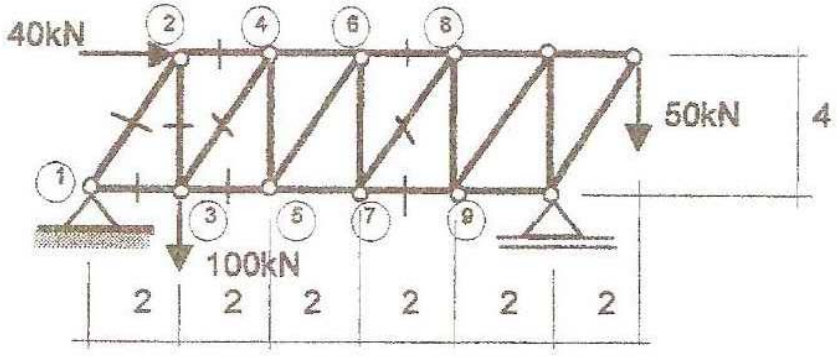

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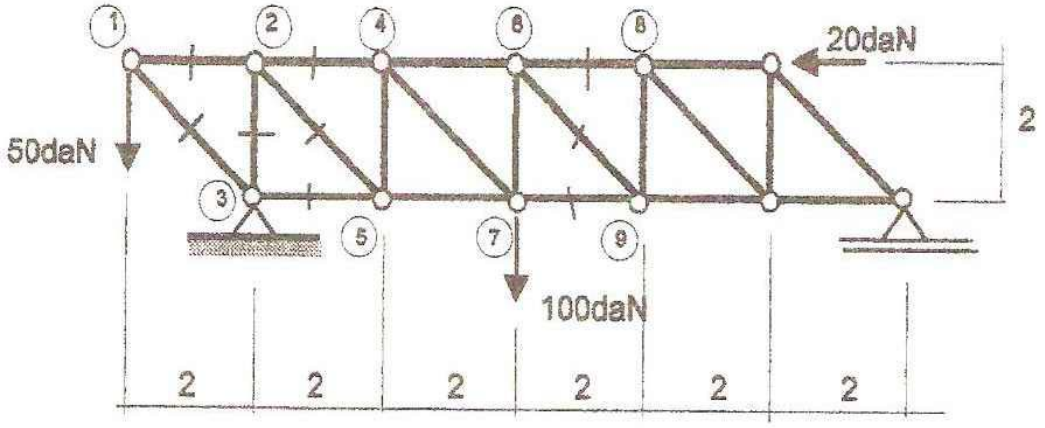

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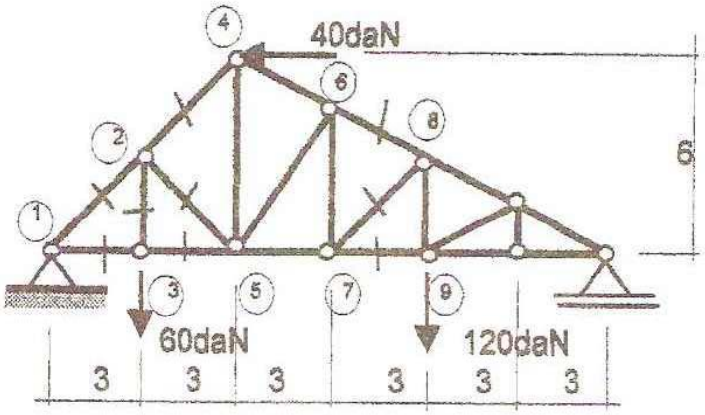

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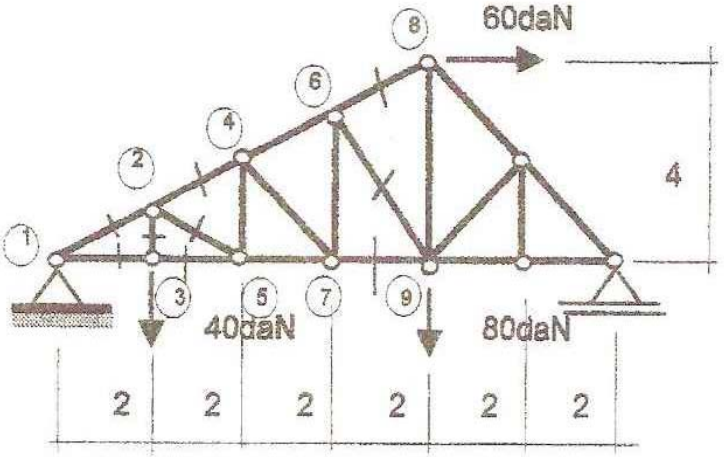

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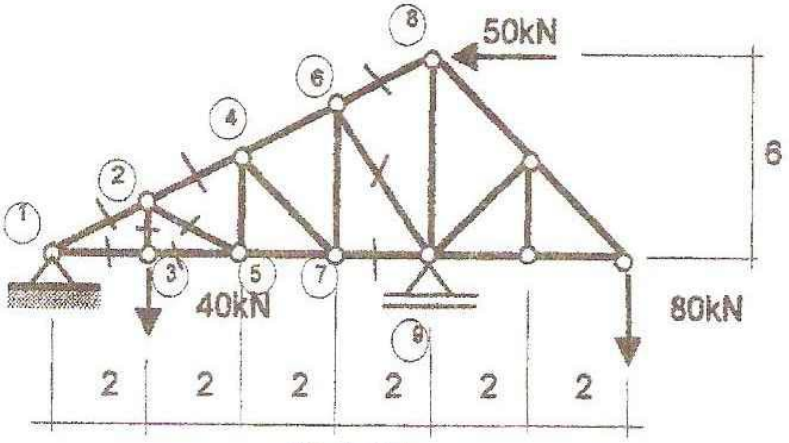

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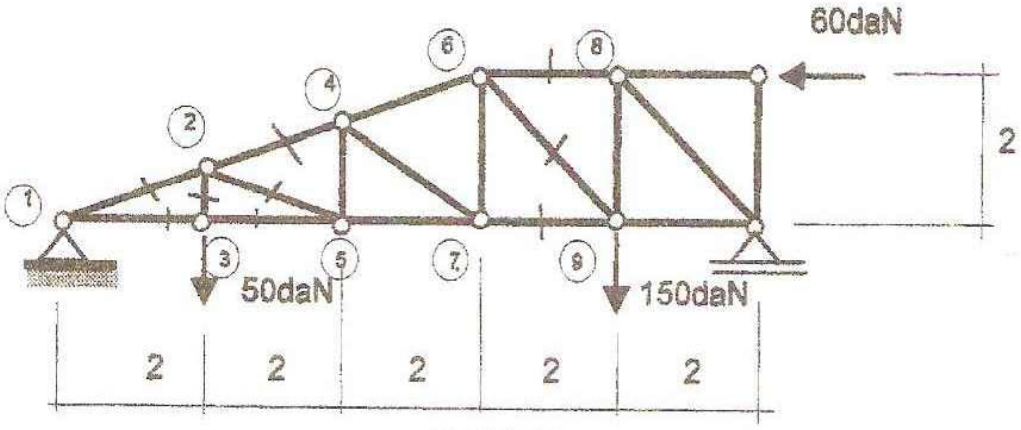

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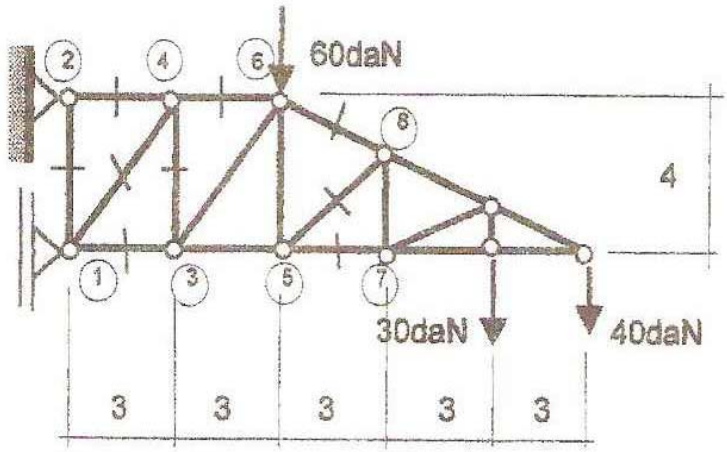

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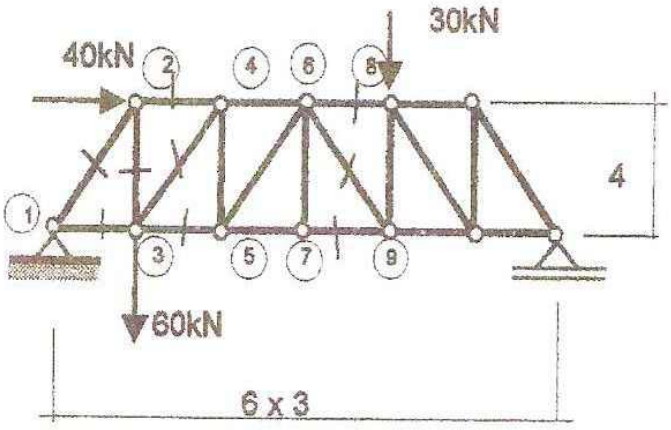

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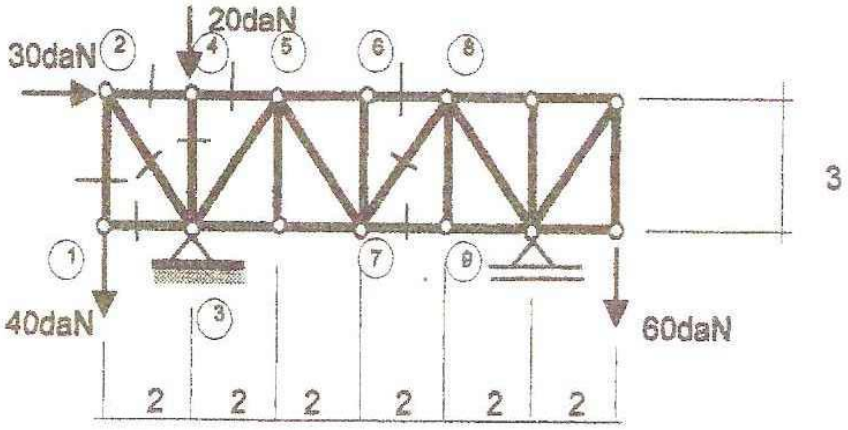


Fig.IV.40.

B.MECANICA II

CAPITOLUL V. CINEMATICA SOLIDULUI RIGID

V.1. MIŞCAREA DE ROTAŢIE PLANĂ

Problema V.1.1.

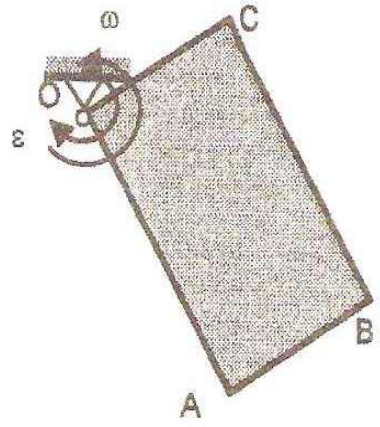
Placa plană din figura V.1.1. se roteşte , în plan vertical, în jurul punctului fix O cu viteza unghiulară ω şi acceleraţia unghiulară ε (având sensurile ca în figură). La momentul t_1 latura OA devine orizontală, moment în care ω are valoarea ω_1 iar ε are valoarea ε_1. Se cere ca pentru momentul t_1:

a)să se calculeze şi reprezinte vitezele vârfurilor dreptunghiului;
b)să se reprezinte distribuţia de viteze pe laturile dreptunghiului;
c)să se calculeze şi reprezinte acceleraţia punctului A.

Problema V.1.2.

Placa plană din figura V.1.2. se roteşte , în plan vertical, în jurul punctului fix O cu viteza unghiulară ω şi acceleraţia unghiulară ε (având sensurile ca în figură). La momentul t_1 latura AB devine verticală, moment în care ω are valoarea ω_1 iar ε are valoarea ε_1. Pentru momentul t_1:

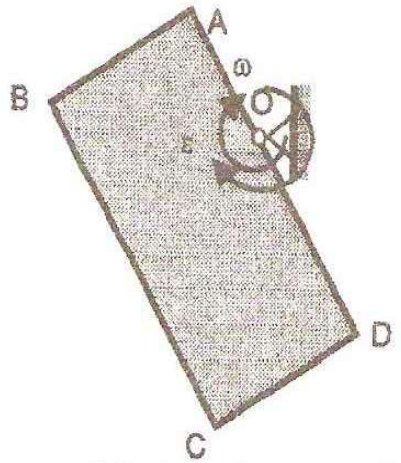
a)să se calculeze şi reprezinte vitezele vârfurilor dreptunghiului;
b)să se reprezinte distribuţia de viteze pe laturile dreptunghiului;
c)să se calculeze şi reprezinte acceleraţia punctului A



OA = 50 cm; AB = 20 cm;

t_1 $\begin{cases} \omega \rightarrow \omega_1 = 3\ s^{-1}; \\ \varepsilon \rightarrow \varepsilon_1 = 6\ s^{-2}; \\ AB \rightarrow \text{verticală} \end{cases}$

Fig.V.1.1.



OA= 0,2m;AB = OD = 0,3m

t_1 $\begin{cases} \omega \rightarrow \omega_1 = 2\ s^{-1}; \\ \varepsilon \rightarrow \varepsilon_1 = 5\ s^{-2}; \\ AB \rightarrow \text{verticală} \end{cases}$

Fig.V.1.2.

Problema V.1.3.

Bara OAB din figura V.1.3. , de forma unui echer dreptunghic, efectuează o rotație în plan vertical în jurul puctului fix O. La momentul t_1 viteza unghiulară are valoarea $\omega_1 = 2$ rad/s, iar accelerația unghiulară are valoarea $\varepsilon_1 = 4$ rad/s^2 (cu sensurile reprezentate în figura V.1.3.) . Se cunosc dimensiunile barei și faptul că în momentul indicat latura OA este orizontală. Pentru momentul t_1 :

a)să se calculeze și reprezinte vitezele punctelor A și B;

b)să se reprezinte distribuția de viteze pe OA și AB;

c)să se calculeze și reprezinte accelerația punctului A.

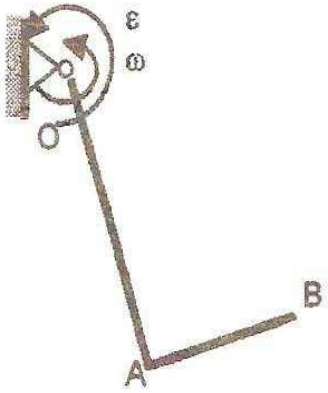
Problema V.1.4.

Bara OAB din figura V.1.4. , de forma unui teu dreptunghic, efectuează o rotație în plan vertical în jurul puctului fix O. La momentul t_1 viteza unghiulară are valoarea $\omega_1 = 5$ rad/s, iar accelerația unghiulară are valoarea $\varepsilon_1 = 6$ rad/s^2 (cu sensurile reprezentate în figura V.1.4.) . Se cunosc dimensiunile barei și faptul că în momentul indicat latura OA este verticală. La momentul t_1 :

a)să se calculeze și reprezinte vitezele punctelor A , B și C;

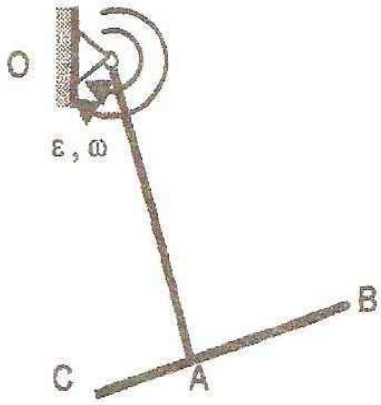
b)să se reprezinte distribuția de viteze pe OA și BC;

c)să se calculeze și reprezinte accelerația punctului A.



OA = 0,6m ; AB = 0,4m

Fig.V.1.3.



OA = 45cm; AB = 30cm;
AC = 20cm.

Fig.V.1.4.

Problema V.1.5.

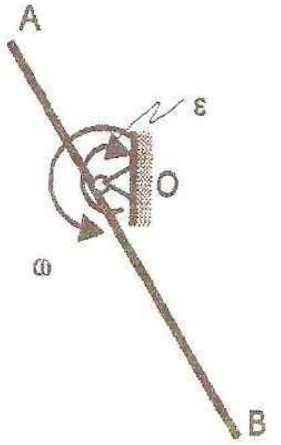
Bara rectilinie AOB din figura V.1.5. efectuează o rotaţie în plan vertical în jurul puctului fix O. La momentul t_1 viteza unghiulară are valoarea $\omega_1 = 3$ rad/s, iar acceleraţia unghiulară are valoarea $\varepsilon_1 = 4$ rad/s^2 (cu sensurile reprezentate în figura V.1.5.) . Se cunosc dimensiunile barei şi faptul că în momentul arătat bara AOB este orizontală. Pentru momentul t_1 :

a)să se calculeze şi reprezinte vitezele punctelor A şi B;
b)să se reprezinte distribuţia de viteze pe AB;
c)să se calculeze şi reprezinte acceleraţia punctului A.

Problema V.1.6.

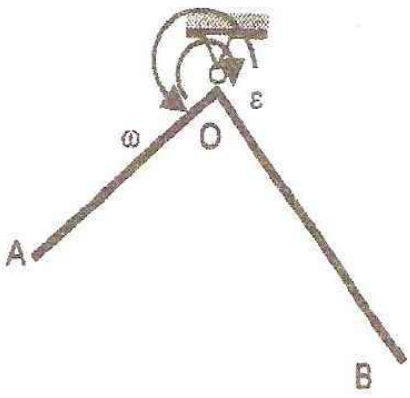
Bara AOB din figura V.1.6., de forma unui echer dreptunghic, efectuează o rotaţie în plan vertical în jurul puctului fix O. La momentul t_1 viteza unghiulară are valoarea $\omega_1 = 4$ rad/s, iar acceleraţia unghiulară are valoarea $\varepsilon_1 = 4$ rad/s^2 (cu sensurile reprezentate în figura V.1.6.) . Se cunosc dimensiunile barei şi faptul că în momentul indicat latura OA este verticală. Să se calculeze şi reprezinte la momentul t_1 :

a) vitezele punctelor A , şi B;
b) distribuţia de viteze pe OA şi OB;
c) acceleraţia punctului A.



OA = 0,6 m ; OB = 0,8 m

Fig.V.1.5.



OA = 0,5m ; OB = 0,7m

Fig.V.1.6.

Problema V.1.7.

Discul de rază R din figura V.1.7. se roteşte în plan vertical în jurul centrului său care este un punct fix. La momentul t_1 viteza unghiulară are valoarea ω_1 = 4 rad/s, acceleraţia unghiulară are valoarea $\varepsilon_1 = 5$ rad/s^2 (cu sensurile reprezentate în figura V.1.7.) , iar diametrul AB este orizontal. La momentul t_1:

a)să se calculeze şi reprezinte vitezele punctelor A , B, C şi D;

b)să se reprezinte distribuţia de viteze pe două diametre perpendiculare (de exemplu AB şi CD);

c)să se calculeze şi reprezinte acceleraţia punctului A.

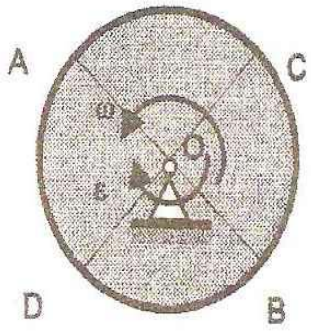
Problema V.1.8.

Discul cu rază R din figura V.1.8. efectuează o rotaţie în plan vertical în jurul puctului fix O. La momentul t_1 viteza unghiulară devine ω_1 = 2 rad/s, iar acceleraţia unghiulară $\varepsilon_1 = 5$ rad/s^2 (cu sensurile reprezentate în figura V.1.8.) . La momentul arătat diametrul OA devine vertical. Pentru momentul indicat :

a)să se calculeze şi reprezinte vitezele punctelor A , B , C şi D;

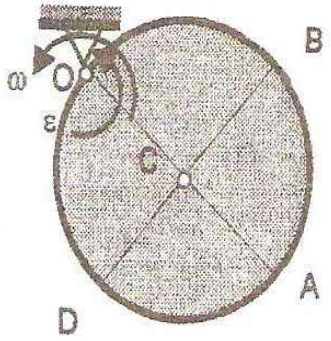
b)să se reprezinte distribuţia de viteze pe cele două diametre perpendiculare: OA şi BD;

c)să se calculeze şi reprezinte acceleraţia punctului A.



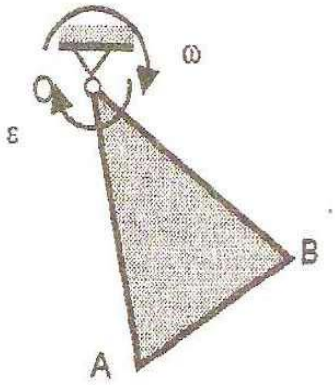
R=50cm

Fig.V.1.7.



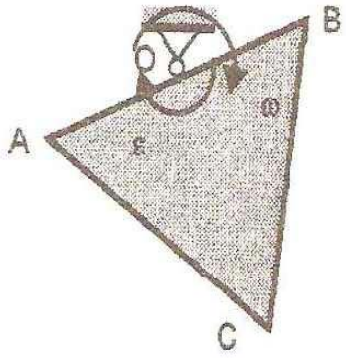
R=0,20m

Fig.V.1.8.



OA=OB=60cm; AB=40cm

Fig.V.1.9.



l = 0,30m

Fig.V.1.10.

Problema V.1.9.

Triunghiul isoscel OAB din figura V.1.9. se roteşte în plan vertical în jurul vârfului său O care este un punct fix. La momentul t_1 viteza unghiulară are valoarea $\omega_1 = 5$ rad/s, acceleraţia unghiulară are valoarea $\varepsilon_1 = 3$ rad/s^2 (cu sensurile reprezentate în figura V.1.9.) , iar latura AB este orizontală. La momentul t_1:

a)să se calculeze şi reprezinte vitezele punctelor A şi B;

b)să se reprezinte distribuţia de viteze pe laturile OA şi AB;

c)să se calculeze şi reprezinte acceleraţia punctului A.

Problema V.1.10.

Triunghiul echilateral de latură *l* din figura V.1.10. efectuează o rotație în plan vertical în jurul punctului fix O (situat la jumătatea laturii AB). La momentul t_1 viteza unghiulară devine $\omega_1 = 4$ rad/s, iar accelerația unghiulară $\varepsilon_1 = 4$ rad/s^2 (cu sensurile reprezentate în figura V.1.10.) . La momentul arătat latura AB devine verticală. Pentru momentul indicat :

a)să se calculeze şi reprezinte vitezele punctelor A , B şi C;
b)să se reprezinte distribuția de viteze pe laturile AB şi BC;
c)să se calculeze şi reprezinte accelerația punctului A.

Problema V.1.11.

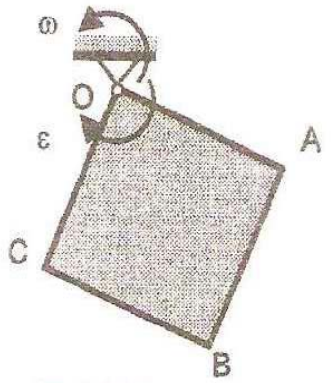
Pătratul de latură *l* din figura V.1.11. efectuează o rotație în plan vertical în jurul punctului fix O. La momentul t_1 viteza unghiulară devine $\omega_1 = 2$ rad/s, iar accelerația unghiulară $\varepsilon_1 = 1$ rad/s^2 (cu sensurile reprezentate în figura V.1.11.) . La momentul arătat latura AB devine verticală. Pentru momentul indicat :

a)să se calculeze şi reprezinte vitezele punctelor A , B şi C;
b)să se reprezinte distribuția de viteze pe laturile OA şi AB;
c)să se calculeze şi reprezinte accelerația punctului A.

Problema V.1.12.

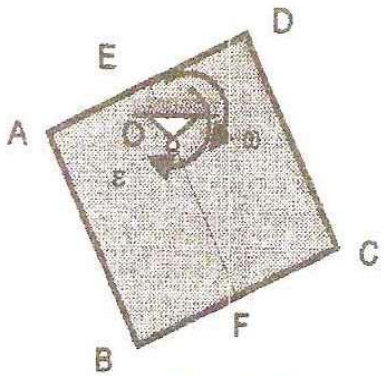
Pătratul de latură 2*l* din figura V.1.12. efectuează o rotație în plan vertical în jurul punctului fix O . La momentul t_1 viteza unghiulară devine $\omega_1 = 3$ rad/s, iar accelerația unghiulară $\varepsilon_1 = 6$ rad/s^2 (cu sensurile reprezentate în figura V.1.12.) . La momentul arătat latura AB devine orizontală. Pentru momentul indicat :

a)să se calculeze şi reprezinte vitezele punctelor A , B , C şi D;
b)să se reprezinte distribuția de viteze pe segmentul EF şi AB;
c)să se calculeze şi reprezinte accelerația punctului A.



l = 0,60m

Fig.V.1.11.



AE=ED=BF=FC=*l* = 20cm;
EO = 10cm

Fig.V.1.12.

Problema V.1.13.

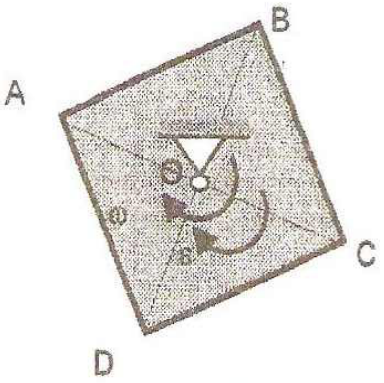
Pătratul de latură *l* din figura V.1.13. efectuează o rotație în plan vertical în jurul punctului fix O (centrul pătratului). La momentul t_1 viteza unghiulară devine ω_1 = 1,5 rad/s, iar accelerația unghiulară ε_1 = 2,5 rad/s^2 (cu sensurile reprezentate în figura V.1.13.) . La momentul arătat latura AB devine verticală. Pentru momentul indicat :

a)să se calculeze şi reprezinte vitezele punctelor A , B , C şi D;
b)să se reprezinte distribuția de viteze pe laturile AB şi AD;
c)să se calculeze şi reprezinte accelerația punctului A.

Problema V.1.14.

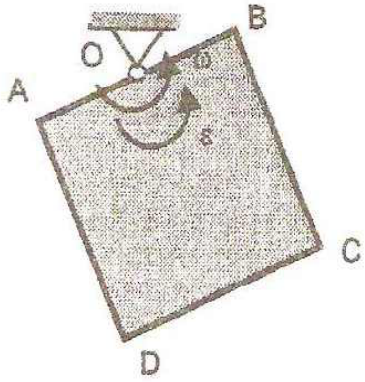
Pătratul de latură 2*l* din figura V.1.14. efectuează o rotație în plan vertical în jurul punctului fix O (mijlocul laturii AB) . La momentul t_1 viteza unghiulară devine ω_1 = 5 rad/s, iar accelerația unghiulară ε_1 = 20 rad/s^2 (cu sensurile reprezentate în figura V.1.14.) . La momentul arătat latura AB devine orizontală. Pentru momentul indicat :

a)să se calculeze şi reprezinte vitezele punctelor A , B , C şi D;
b)să se reprezinte distribuția de viteze pe laturile AB şi BC;
c)să se calculeze şi reprezinte accelerația punctului A.



l = 40cm

Fig.V.1.13.



l = AO = OB = 0,2m

Fig.V.1.14.

Problema V.1.15.

Dreptunghiul din figura V.1.15. efectuează o rotație în plan vertical în jurul punctului fix O (centrul dreptunghiului). La momentul t_1 viteza unghiulară devine ω_1 = 3,5 rad/s, iar accelerația unghiulară ε_1 = 2,5 rad/s^2 (cu sensurile reprezentate în figura V.1.15.) . La momentul arătat latura AB devine verticală. Pentru momentul indicat :

a)să se calculeze şi reprezinte vitezele punctelor A , B , C şi D;

b)să se reprezinte distribuția de viteze pe diagonala AC și latura AB;

c)să se calculeze și reprezinte accelerația punctului A.

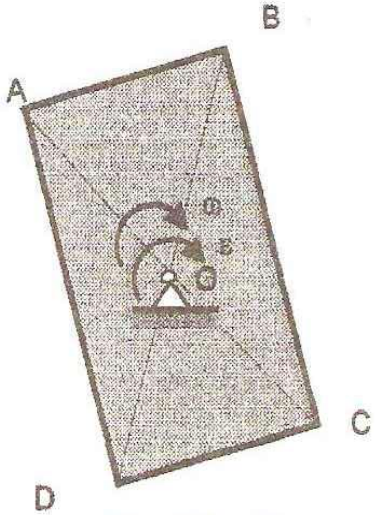
Problema V.1.16.

Dreptunghiul din figura V.1.16. efectuează o rotație în plan vertical în jurul punctului fix O (mijlocul laturii AB) . La momentul t_1 viteza unghiulară devine ω_1 = 4 rad/s, iar accelerația unghiulară ε_1 = 2 rad/s^2 (cu sensurile reprezentate în figura V.1.16.) . La momentul arătat latura AB devine orizontală. Pentru momentul indicat :

a)să se calculeze și reprezinte vitezele punctelor A , B , C și D;

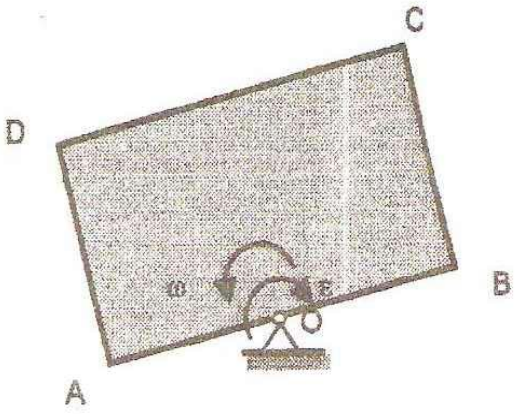
b)să se reprezinte distribuția de viteze pe laturile AB și BC;

c)să se calculeze și reprezinte accelerația punctului A.



AB = DC = 40cm ;
AD = BC = 70cm

Fig.V.1.15.



AO = OB = 30 cm;
AD = BC = 40 cm.

Fig.V.1.16.

Problema V.1.17.

Triunghiul dreptunghic din figura V.1.17. efectuează o rotație în plan vertical în jurul punctului fix O . La momentul t_1 viteza unghiulară devine ω_1 = 3 rad/s, iar accelerația unghiulară ε_1 = 2,5 rad/s^2 (cu sensurile reprezentate în figura V.1.17.) . La momentul arătat latura OA devine verticală. Pentru momentul indicat :

a)să se calculeze și reprezinte vitezele punctelor A și B;

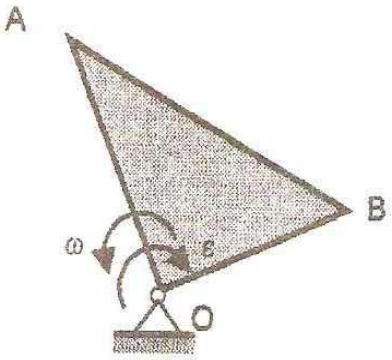
b)să se reprezinte distribuția de viteze pe cateta OA și ipotenuza AB;

c)să se calculeze și reprezinte accelerația punctului A.

Problema V.1.18.

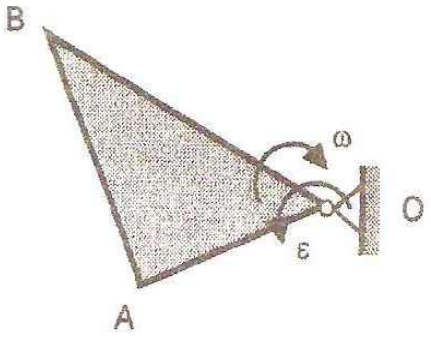
Triunghiul dreptunghic din figura V.1.18. efectuează o rotație în plan vertical în jurul punctului fix O . La momentul t_1 viteza unghiulară devine $\omega_1 = 3$ rad/s, iar accelerația unghiulară $\varepsilon_1 = 8$ rad/s^2 (cu sensurile reprezentate în figura V.1.18.) . La momentul arătat cateta AB devine verticală. Pentru momentul indicat :

a)să se calculeze şi reprezinte vitezele punctelor A şi B;
b)să se reprezinte distribuția de viteze pe cele două catete;
c)să se calculeze şi reprezinte accelerația punctului A.



OA = 80cm; OB = 60 cm

Fig.V.1.17.



OA =0,3m ; AB =0,4m

Fig.V.1.18.

Problema V.1.19.

Triunghiul isoscel din figura V.1.19. efectuează o rotație în plan vertical în jurul punctului fix O (centrul de greutate al triunghiului) . La momentul t_1 viteza unghiulară devine $\omega_1 = 5$ rad/s, iar accelerația unghiulară $\varepsilon_1 = 25$ rad/s^2 (cu sensurile reprezentate în figura V.1.19.) . La momentul arătat latura AB devine verticală. Pentru momentul indicat :

a)să se calculeze şi reprezinte vitezele punctelor A , B şi C;
b)să se reprezinte distribuția de viteze pe înălțimea DC şi latura AB;
c)să se calculeze şi reprezinte accelerația punctului A.

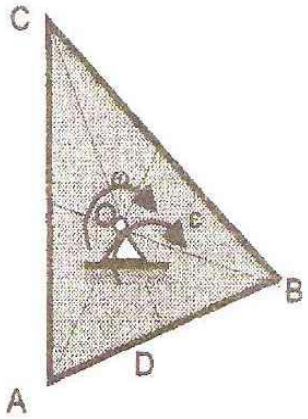
Problema V.1.20.

Discul de rază R din figura V.1.20. efectuează o rotație în plan vertical în jurul punctului fix O (mijlocul razei CA) . La momentul t_1 viteza unghiulară devine $\omega_1 = 2$ rad/s, iar accelerația unghiulară $\varepsilon_1 = 8$ rad/s^2 (cu sensurile reprezentate în figura V.1.20.) . La momentul arătat diametrul AC devine vertical. Pentru momentul indicat :

a)să se calculeze şi reprezinte vitezele punctelor A , B, C , D şi E;

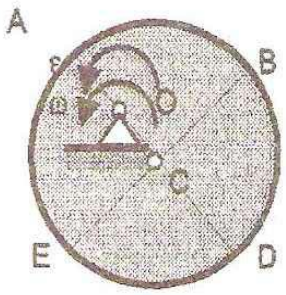
b)să se reprezinte distribuţia de viteze pe cele două diametre perpendiculare: AOCD şi BCE;

c)să se calculeze şi reprezinte acceleraţia punctului D.



AC = BC = 50cm ;
AD = DB = 30 cm.

Fig.V.1.19.



AO = OC;
R = 0,6m

Fig.V.1.20.

V.2.MIŞCAREA PLAN PARALELĂ (ÎN PLAN)

Problema V.2.1.

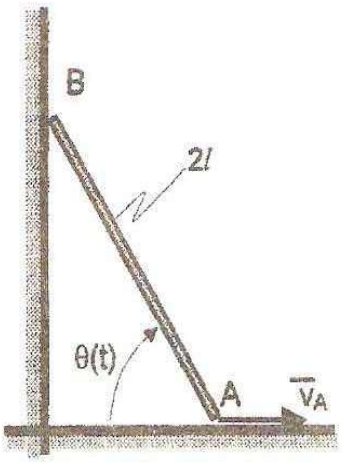
Bara AB de lungime 2*l* (Fig.V.2.1.) se mişcă astfel încât cele două extremităţi A şi B descriu două drepte ortogonale (una orizontală şi alta verticală). Extremitatea A are viteza constantă: v_A = 10 cm /s . La momentul t_1 bara face cu orizontala unghiul de 60°. Pentru momentul indicat să se calculeze şi reprezinte:

a) viteza punctului B şi viteza minimă pe bară;
b) distribuţia de viteze pe bara AB;
c) acceleraţia punctului B.

Problema V.2.2.

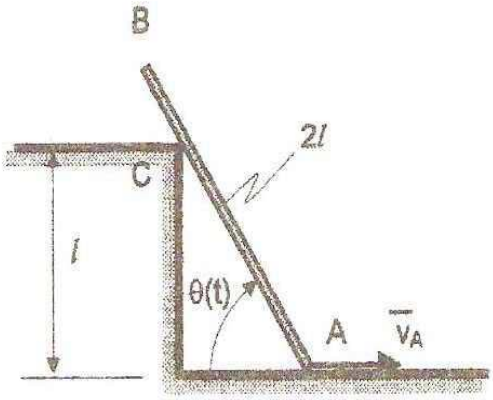
Bara AB de lungime 2*l* se mişcă astfel încât extremitatea A descrie o dreaptă orizontală (Fig.V.2.2.) cu viteza constantă: v_A = 20 cm /s şi se reazemă de punctul fix C . La momentul t_1 bara face cu orizontala unghiul de 45°. Pentru momentul t_1 să se calculeze şi reprezinte:

a) vitezele punctelor B şi C;
b) distribuţia de viteze pe bara AB;
c) acceleraţia punctului B.



l = 10cm

Fig.V.2.1.



l = 10cm

Fig.V.2.2.

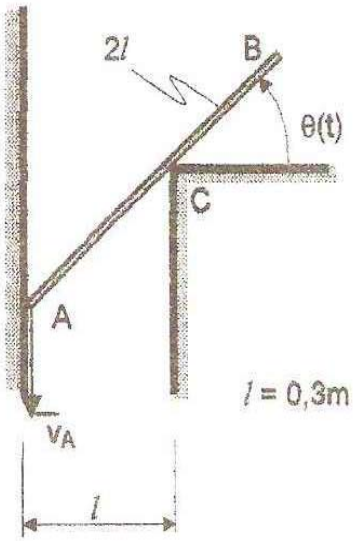
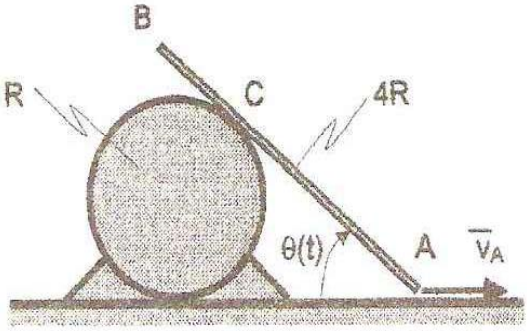


Fig.V.2.3.



R = 30cm

Fig.V.2.4.

Problema V.2.3.

Bara AB de lungime 2l se mişcă astfel încât extremitatea A descrie o dreaptă verticală (Fig.V.2.3.) cu viteza constantă: v_A = 0,50 m /s şi se reazemă de punctul fix C . La momentul t_1 bara face cu verticala unghiul de 45°. Pentru momentul t_1 să se calculeze şi reprezinte:

a) vitezele punctelor B şi C;
b) distribuţia de viteze pe bara AB;
c) acceleraţia punctului B.

Problema V.2.4.

Bara AB de lungime 4R se mişcă astfel încât extremitatea A descrie o dreaptă orizontală (Fig.V.2.4.) cu viteza constantă: v_A = 20 cm /s şi este tangentă în punctul C la cercul fix vertical de rază R. La momentul t_1 bara face cu orizontala unghiul de $\pi/3$. Pentru momentul t_1 să se calculeze şi reprezinte:

a) vitezele punctelor B şi C;
b) distribuţia de viteze pe bara AB;
c) acceleraţia punctului B.

Problema V.2.5.

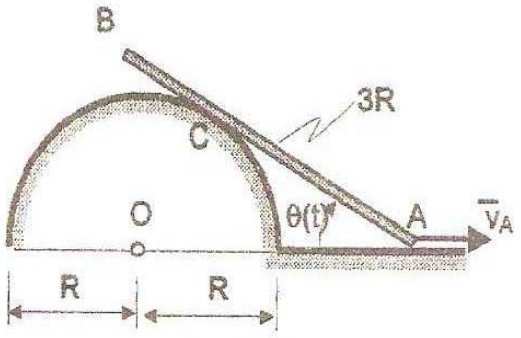
Bara AB de lungime 3R se mişcă astfel încât extremitatea A descrie o dreaptă orizontală, ce trece prin centrul O al cercului vertical fix de rază R (Fig.V.2.5.), cu viteza constantă: v_A = 0,40 m /s şi se reazemă de acel cerc fix în punctul C (punct de tangenţă) . La momentul t_1 bara face cu orizontala unghiul de 30°. Pentru momentul t_1 să se calculeze şi reprezinte:

a) vitezele punctelor B şi C;
b) distribuţia de viteze pe bara AB;
c) acceleraţia punctului B.

Problema V.2.6.

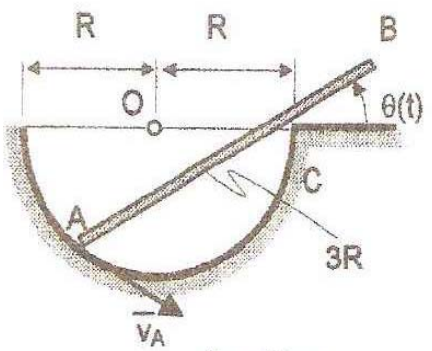
Bara AB de lungime 3R se mişcă astfel încât extremitatea A descrie cercul vertical fix de rază R (Fig.V.2.6.) cu viteza constantă: v_A = 50 cm /s şi se sprijină de punctul fix C (extremitatea diametrului orizontal al cercului). La momentul t_1 bara face cu orizontala unghiul de $\pi/4$. Pentru momentul t_1 să se calculeze şi reprezinte:

a) vitezele punctelor B şi C;
b) distribuţia de viteze pe bara AB;
c) acceleraţia punctului B.



R = 0,25m

Fig.V.2.5.



R = 20cm

Fig.V.2.6.

Problema V.2.7.

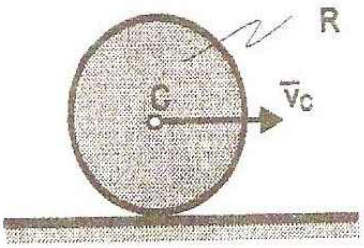
Discul de rază R = 30 cm se rostogoleşte, în plan vertical, fără să alunece pe o dreaptă orizontală (Fig.V.2.7.). Cunoscând faptul că viteza centrului discului este constantă şi are mărimea v_C = 60 cm/s, să se calculeze şi reprezinte la un moment oarecare al mişcării:

a) vitezele extremităţilor diametrelor perpendiculare, orizontală şi verticală;
b) distribuţia de viteze pe cele două diametre;
c) acceleraţia uneia din extremităţile diametrului orizontal.

Problema V.2.8.

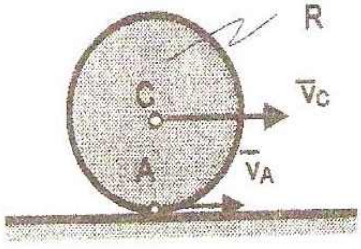
Discul de rază R = 0,50 m se rostogoleşte şi alunecă, în plan vertical, pe o dreaptă orizontală (Fig.V.2.8.). Cunoscând faptul că viteza centrului discului este constantă şi are mărimea v_C = 1,5 m/s iar viteza de alunecare a discului pe dreapta orizontală este v_A = 0,5 m/s şi este de asemenea constantă, să se calculeze şi reprezinte la un moment oarecare al mişcării:

a) vitezele extremităţilor diametrelor perpendiculare, orizontală şi verticală;
b) distribuţia de viteze pe cele două diametre;
c) acceleraţia uneia din extremităţile diametrului orizontal.



rostogolire fără alunecare

Fig.V.2.7.



rostogolire cu alunecare

Fig.V.2.8.

Problema V.2.9.

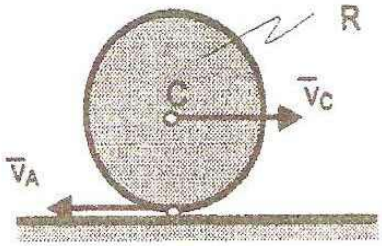
Discul de rază R = 10 cm se rostogoleşte şi patinează, în plan vertical, pe o dreaptă orizontală (Fig.V.2.9.). Cunoscând faptul că viteza centrului discului este constantă şi are mărimea v_C = 15 cm/s iar viteza de alunecare înspre înapoi a discului pe dreapta orizontală este v_A = 15 cm/s şi este de asemenea constantă, să se calculeze şi reprezinte la un moment oarecare al mişcării:

a) vitezele extremităţilor diametrelor perpendiculare, orizontală şi verticală;
b) distribuţia de viteze pe cele două diametre;
c) acceleraţia uneia din extremităţile diametrului orizontal.

Problema V.2.10.

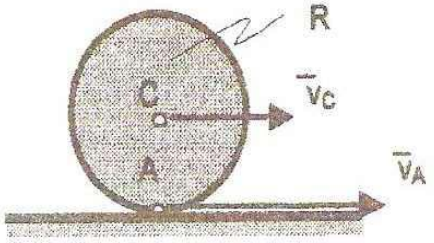
Discul de rază R = 20 cm se rostogoleşte şi alunecă, în plan vertical, pe o dreaptă orizontală (Fig.V.2.10.). Cunoscând faptul că viteza centrului discului este constantă şi are mărimea v_C = 30 cm/s iar viteza de alunecare a discului pe dreapta orizontală este v_A = 60 cm/s şi este de asemenea constantă, să se calculeze şi reprezinte la un moment oarecare al mişcării:

a) vitezele extremităţilor diametrelor perpendiculare, orizontală şi verticală;
b) distribuţia de viteze pe cele două diametre;
c) acceleraţia uneia din extremităţile diametrului orizontal.



rostogolire cu patinare

Fig.V.2.9.



rostogolire cu alunecare

Fig.V.2.10.

Problema V.2.11.

Bara AB de lungime R = 20 cm se mişcă în interiorul unui cerc vertical fix de rază R rezemându-se cu cele două extremităţi A şi B pe cerc (Fig.V.2.11.). Viteza extremităţii A este constantă şi egală cu v_A = 60 cm/s. La momentul când bara este orizontală să se calculeze şi reprezinte:

a) viteza punctului B şi viteza mijlocului barei;
b) distribuţia de viteze pe bară;
c) acceleraţia punctului B.

Problema V.2.12.

Bara ABC de forma unui echer dreptunghic se mişcă astfel încât vârful B descrie cercul vertical fix de rază R = 0,4m (Fig.V.2.12.) cu viteza constantă v_B = 0,2 m/s. Bara se reazemă în acelaşi timp în punctele fixe D şi E (extremităţile diametrului orizontal al cercului de rază R). La momentul t_1 al mişcării punctul B se află pe verticala centrului O al cercului. Pentru acest moment să se calculeze şi reprezinte:

a) vitezele punctelor A, C, D şi E;
b) distribuţia de viteze pe cele două laturi ale echerului :AB şi BC;
c) acceleraţia punctului A.

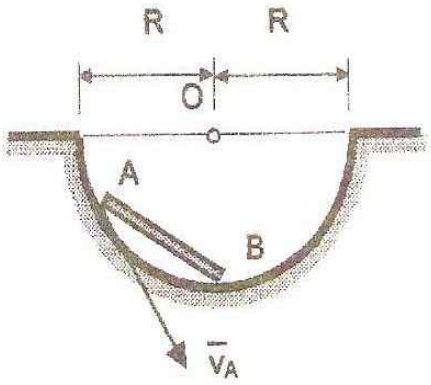
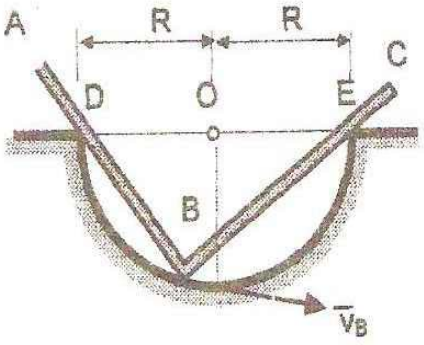
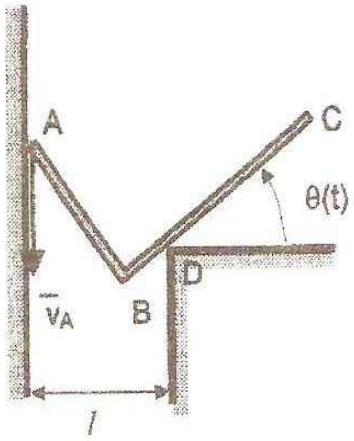


Fig.V.2.11.



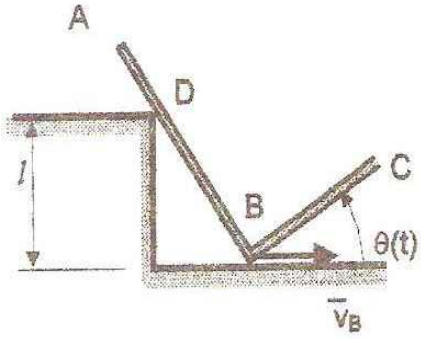
AB = 2R, BC = 3R

Fig.V.2.12.



AB = l = 0,6m, BC = 2l

Fig.V.2.13.



AB = 2l, BC = l = 0,5m

Fig.V.2.14.

Problema V.2.13.

Echerul dreptunghic ABC se mişcă în planul vertical descriind cu extremitatea A o dreaptă fixă verticală şi rezemându-se în punctul fix D (Fig.V.2.13.). Cunoscând faptul că viteza punctului A este constantă: v_A = 5m/s, să se calculeze şi reprezinte la momentul t_1 (când unghiul dintre latura AB şi verticală este $\pi/4$):

a) vitezele punctelor B, C şi D;
b) distribuţia de viteze pe cele două laturi ale echerului;
c) acceleraţia punctului B.

Problema V.2.14.

Echerul dreptunghic ABC se mişcă în planul vertical descriind cu vârful B o dreaptă fixă orizontală şi se reazemă în punctul fix D (Fig.V.2.14.). Cunoscând

faptul că viteza punctului B este constantă: v_B = 0,8m/s, să se calculeze şi reprezinte la momentul t_1 (când unghiul dintre latura AB şi verticală este $\pi/6$):

a) vitezele punctelor A, C şi D;
d) distribuţia de viteze pe cele două laturi ale echerului;
d) acceleraţia punctului B.

Problema V.2.15.

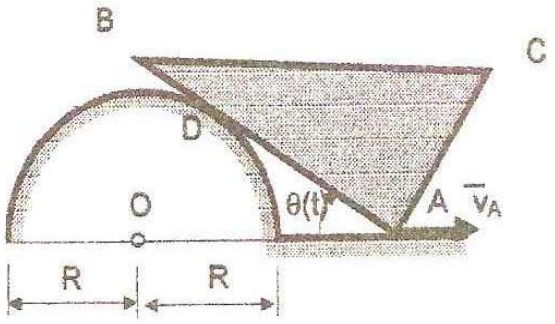
Triunghiul dreptunghic ABC (unghiul drept în A) se reazemă în D de un cerc vertical fix cu raza R = 0,6m , mişcându-se în plan vertical, şi descrie cu vârful A dreapta fixă orizontală ce trece prin centrul O al cercului (Fig.V.2.15.). Viteza punctului A este constantă şi egală cu : v_A = 0,3m/s. La momentul t_1 al mişcării cateta AB face cu orizontala unghiul de $\pi/4$. Pentru momentul acesta se cere să se calculeze şi reprezinte:

a) vitezele punctelor B, C şi D;
b) distribuţia de viteze pe laturile triunghiului;
c) acceleraţia punctului C.

Problema V.2.116.

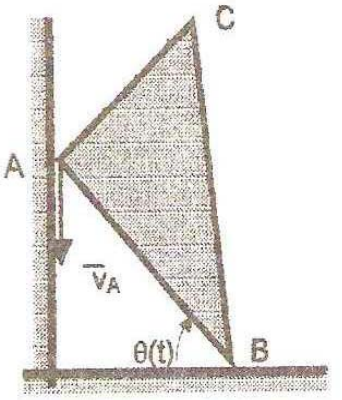
Triunghiul dreptunghic ABC (unghiul drept în A) se mişcă în plan vertical şi descrie cu vârfurile A şi B două drepte fixe (una verticală şi una orizontală) (Fig.V.2.15.). Viteza punctului A este constantă şi egală cu : v_A = 30 cm/s. La momentul t_1 al mişcării cateta AB face cu verticala unghiul de $\pi/6$. Pentru momentul acesta se cere să se calculeze şi reprezinte:

a) vitezele punctelor B, şi C;
b) distribuţia de viteze pe laturile triunghiului;
c) acceleraţia punctului B.



AB = 3R, AC = 2R

Fig.V.2.15.



AB = 40cm, AC = 30cm

Fig.V.2.16.

Problema V.2.17.

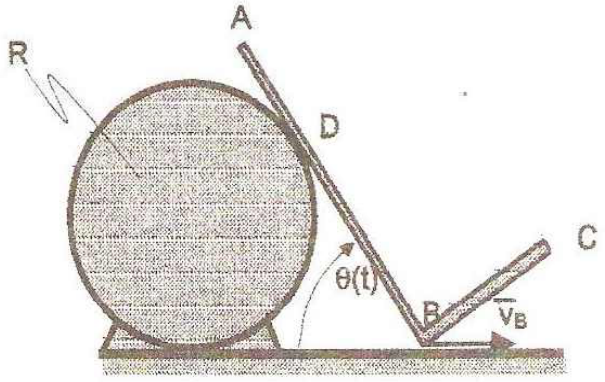
Echerul dreptunghic ABC se mişcă în planul vertical descriind cu vârful B o dreaptă fixă orizontală şi se reazemă în D pe un cerc fix vertical de rază R = 0,4m (Fig.V.2.17.). Cunoscând faptul că viteza punctului B este constantă: v_B = 0,8m/s şi că dreapta orizontală este tangentă la cercul de rază R să se calculeze şi reprezinte la momentul t_1 (când unghiul dintre latura AB şi orizontală este $\pi/3$):

a) vitezele punctelor A, C şi D;
b) distribuţia de viteze pe cele două laturi ale echerului;
c) acceleraţia punctului A.

Problema V.2.18.

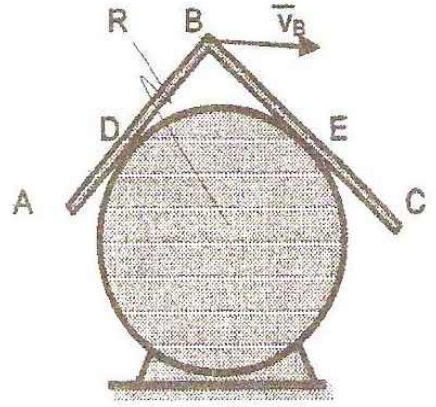
Echerul dreptunghic ABC se mişcă în planul vertical rezemându-se în D şi E pe un cerc fix vertical de rază R = 0,5m (Fig.V.2.18.). Cunoscând faptul că viteza punctului B este constantă: v_B = 0,8m/s să se calculeze şi reprezinte la momentul t_1 (când latura AB devine orizontală):

a) vitezele punctelor A, C, D şi E;
b) distribuţia de viteze pe cele două laturi ale echerului;
c) acceleraţia punctului B.



AB = 4R, BC = R

Fig.V.2.17.



AB = 2R, BC = 3R

Fig.V.2.18.

Problema V.2.19.

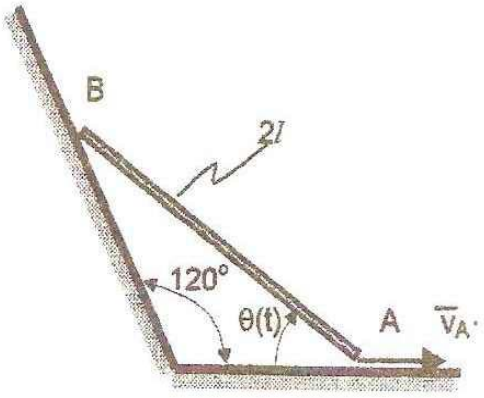
Bara AB de lungime 2*l* se mişcă astfel încât extremitatea A descrie o dreaptă orizontală cu viteza constantă: v_A = 0,50 m /s, iar extremitatea B o dreaptă înclinată ca în figura V.2.19 . La momentul t_1 bara face cu orizontala unghiul de 30°. Pentru momentul t_1 să se calculeze şi reprezinte:

a) viteza punctului B şi a mijlocului barei;
b) distribuţia de viteze pe bara AB;
c) acceleraţia punctului B.

Problema V.2.20.

Bara AB se mişcă în planul vertical descriind cu extremitatea A un cerc fix vertical de rază R = 1,2m , iar cu extremitatea B o dreaptă verticală fixă ce trece prin centrul O al cercului de rază R (Fig.V.2.20.). Viteza puntului A este constantă , v_A = 2 m/s. La momentul t_1 bara face cu verticala unghiul de 60°. Pentru momentul indicat să se calculeze şi reprezinte:

a) viteza punctului B şi viteza mijlocului barei;
b) distribuţia de viteze pe bară;
c) acceleraţia punctului B.



l = 0,6m

Fig.V.2.19.

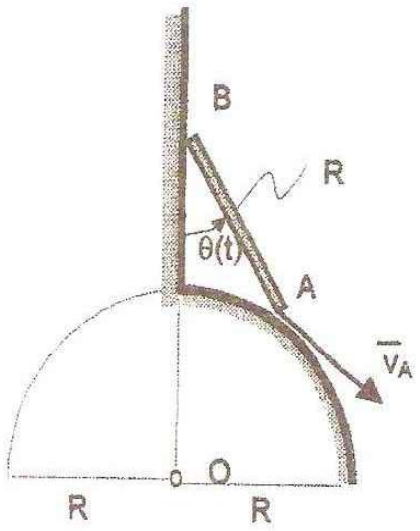
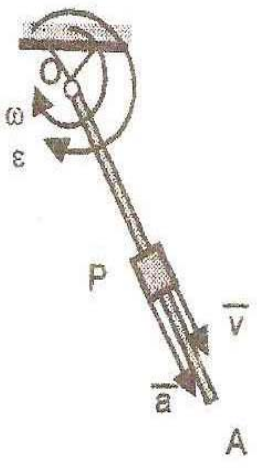


Fig.V.2.20.

CAPITOLUL VI. MIŞCAREA RELATIVĂ A PUNCTULUI

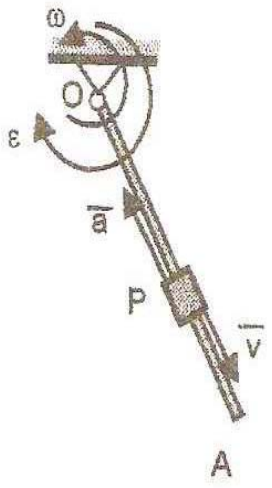
Problemele VI.1., VI.2.

Culisa P poate aluneca cu viteza $\overline{v}$ şi acceleraţia $\overline{a}$ (faţă de bară) pe bara OA ce efectuează o rotaţie în plan vertical în jurul punctului fix O cu viteza unghiulară ω şi acceleraţia unghiulară ε. La momentul t_1 culisa ajunge în capătul A, bara devine orizontală, viteza $\overline{v}$, acceleraţia $\overline{a}$, viteza unghiulară ω şi acceleraţia unghiulară ε vor avea valorile: v_1, a_1, ω_1 respectiv ε_1 şi cu sensurile din figurile VI.1 şi VI.2. Pentru momentul t_1 să se calculeze şi reprezinte viteza şi acceleraţia absolută a culisei P.



v_1= 2m/s, a_1 = 3m/s^2, ω_1= 1,5 s^{-1}, ε_1 = 2 s^{-2}, OA = 0,8m.

Fig.VI.1.

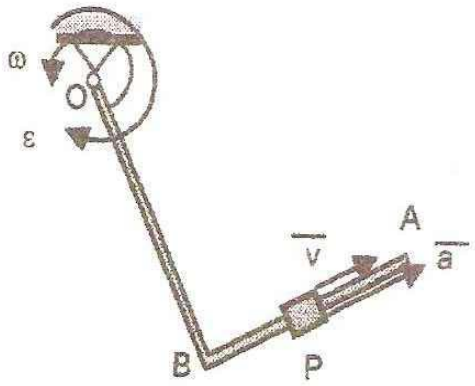


v_1= 1,5m/s, a_1 = 2m/s^2, ω_1= 2 s^{-1}, ε_1 = 3 s^{-2}, OA = 1,2m.

Fig.VI.2.

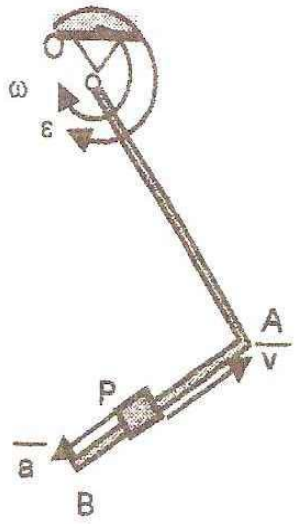
Problemele VI.3., VI.4.

Culisa P alunecă cu viteza $\overline{v}$ şi acceleraţia $\overline{a}$ (faţă de bară) pe latura AB a barei cotite ce efectuează o rotaţie în plan vertical în jurul punctului fix O cu viteza unghiulară ω şi acceleraţia unghiulară ε. La momentul t_1 culisa ajunge în capătul A, bara AB devine orizontală, viteza $\overline{v}$, acceleraţia $\overline{a}$, viteza unghiulară ω şi acceleraţia unghiulară ε vor lua valorile: v_1, a_1, ω_1 respectiv ε_1 şi cu sensurile din figurile VI.3 şi VI.4. Pentru momentul t_1 al mişcării să se calculeze şi reprezinte viteza absolută şi acceleraţia absolută a culisei P.



OB = 40cm, AB = 30cm,
ω_1 = 1 rad/s, ε_1 = 2 rad/s^2,
v_1 = 60cm/s, a_1 = 80 cm/s^2.

Fig.VI.3.



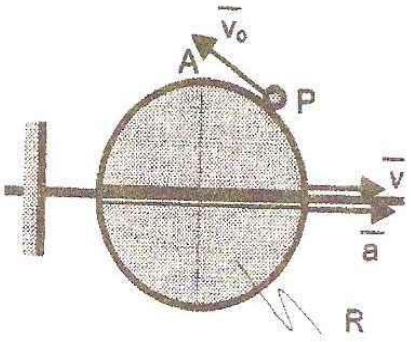
OA = 0,6m, AB = 0,3m,
ω_1 = 2 rad/s, ε_1 = 3 rad/s^2,
v_1 = 0,60m/s, a_1 = 0,80 m/s^2.

Fig.VI.4.

Problemele VI.5., VI.6., VI.7., VI.8.

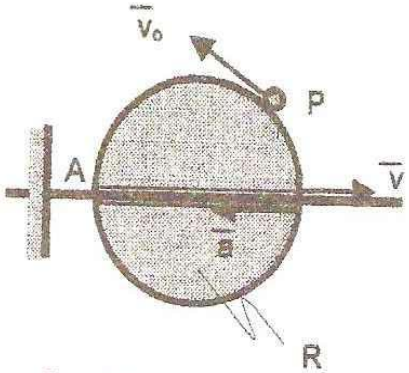
Discul de rază R din figura VI.5., VI.6., VI.7. respectiv VI.8. alunecă de-a lungul diametrului orizontal, în plan vertical cu viteza $\overline{v}$ şi acceleraţia $\overline{a}$. În acelaşi timp pe periferia discului se mişcă, cu viteză constantă $\overline{v}_o$ faţă de disc, un punct material P. La momentul t_1 punctul P ajunge în A, viteza $\overline{v}$ are valoarea v_1, iar acceleraţia $\overline{a}$ are mărimea a_1 şi cu sensurile din figurile mai înainte menţionate.

Pentru momentul indicat să se calculeze şi reprezinte viteza absolută şi acceleraţia absolută a punctului P.



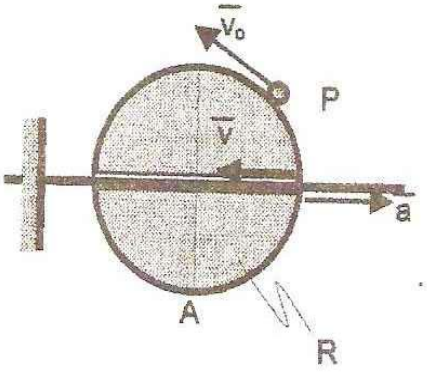
$R = 20$ cm, $v_0 = 40$ cm/s,
$v_1 = 30$cm/s, $a_1 = 60$cm/s^2.

Fig.VI.5.



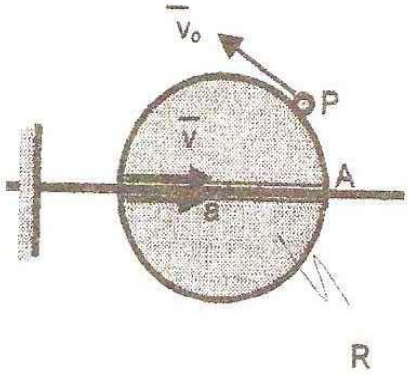
$R = 0,3$m, $v_0 = 0,4$ m/s,
$v_1 = 0,5$m/s, $a_1 = 0,5$m/s^2.

Fig.VI.6.



$R = 25$cm, $v_0 = 60$ cm/s,
$v_1 = 50$ cm/s, $a_1 = 90$cm/s^2.

Fig.VI.7.



$R = 1,2$m, $v_0 = 2$m/s,
$v_1 = 1,5$m/s, $a_1 = 3$m/s^2.

Fig.VI.8.

Problema VI.9.

Discul de rază R = 0,6 m se roteşte uniform în jurul centrului său O cu viteza unghiulară $\omega_0 = 3$ rad/s (Fig.VI.9.). În acelaşi timp pe periferia lui se deplasează cu viteză constantă $\bar{v}_0 = 3$ m/s , faţă de disc , un punct material P. La un moment dat punctul se află în extremitatea superioară a diametrului vertical al discului. Pentru acel moment să se calculeze şi reprezinte viteza şi acceleraţia absolută a punctului P.

Problema VI. 10.

Discul de rază R = 35 cm se roteşte în jurul centrului său O cu viteza unghiulară ω şi acceleraţia unghiulară ε. În acelaşi timp pe periferia lui se deplasează cu viteză constantă $v_0 = 100$cm/s, faţă de disc , un punct material P. La

momentul t_1 punctul se află în extremitatea din stânga a diametrului orizontal al discului, viteza unghiulară luând valoarea ω_1 , iar accelerația unghiulară valoarea ε_1, cu sensurile din figura VI.10. Pentru acel moment să se calculeze și reprezinte viteza și accelerația absolută a punctului P.

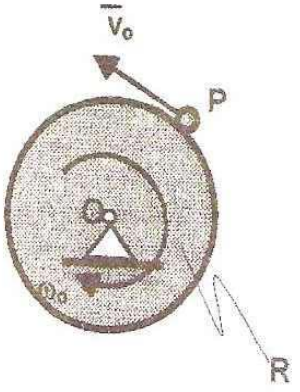
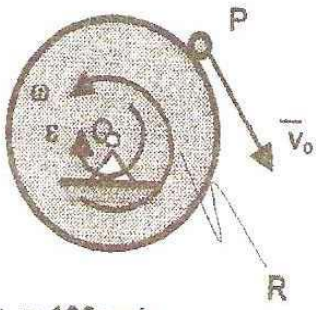


Fig.VI.9.



v_o = 100cm/s

ω_1 = 2 rad/s, ε_1 = 5 rad/s^2.

Fig.VI.10.

Problema VI.11.

Cadrul dreptunghiular din figura VI.11. alunecă , în plan vertical, pe o dreaptă fixă orizontală cu viteza $\bar{v}_A$ și accelerația $\bar{a}_A$. În același timp pe latura verticală AB alunecă o culisă P cu viteza $\bar{v}$ și accelerația $\bar{a}$, în raport cu cadrul. La momentul t_1 culisa ajunge în B iar vitezele și accelerațiile devin: $\bar{v}_{A1}$, $\bar{a}_{A1}$, $\bar{v}_1$ respectiv $\bar{a}_1$ și cu sensurile prezentate în figură. Pentru momentul arătat să se calculeze și reprezinte viteza și accelerația absolută a culisei P.

Problema VI.12.

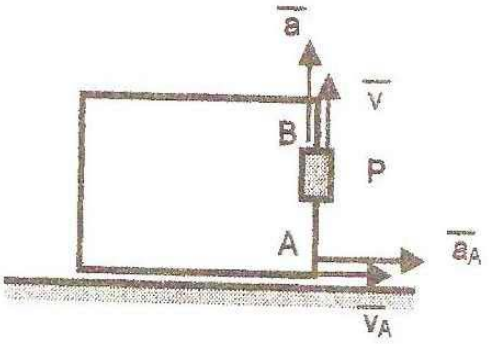
Un cadru de forma unui pătrat alunecă, în plan vertical , pe o dreaptă fixă orizontală cu viteza $\bar{v}_A$ și accelerația $\bar{a}_A$. În același timp , pe diagonala pătratului alunecă o culisă P cu viteza $\bar{v}$ și accelerația $\bar{a}$, în raport cu cadrul. La momentul t_1 culisa ajunge în B iar vitezele și accelerațiile devin: $\bar{v}_{A1}$, $\bar{a}_{A1}$, $\bar{v}_1$ respectiv $\bar{a}_1$ și cu sensurile prezentate în figură. Pentru momentul arătat să se calculeze și reprezinte viteza și accelerația absolută a culisei P.

Problema VI.13.

Discul de rază R = 0,3 efectuează o rotație, în raport cu punctul fix O, în plan vertical, cu viteza unghiulară ω și accelerația unghiulară ε. În același timp pe periferia discului se deplasează un punct material P cu viteza constantă v_o = 5m/s în raport cu discul. La momentul t_1 diametrul OA este vertical , punctul P ajunge în A, viteza unghiulară devine ω_1 = 10 rad/s , iar accelerația unghiulară ε_1 = 50rad/s^2 și cu sensurile din figura VI.13. Pentru acest moment să se calculeze și reprezinte viteza și accelerația absolută a punctului P.

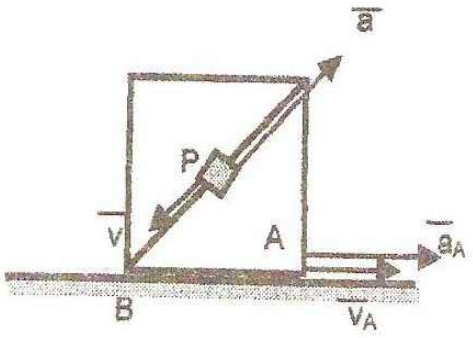
Problema VI.14.

Discul de rază R = 10cm efectuează o rotație, în raport cu punctul fix O, în plan vertical, cu viteza unghiulară ω și accelerația unghiulară ε. În același timp pe periferia discului se deplasează un punct material P cu viteza constantă v_o = 30cm/s în raport cu discul. La momentul t_1 diametrul OA este orizontal , punctul P ajunge în B (extremitatea diametrului perpendicular pe OA), viteza unghiulară devine ω_1 = 3 rad/s , iar accelerația unghiulară ε_1 = 10rad/s^2 și cu sensurile reprezentate în figura VI.14. Pentru acest moment să se calculeze și reprezinte viteza și accelerația absolută a punctului P.



v_{A1} = 30cm/s, a_{A1} = 40cm/s^2,
v_1 = 50cm/s, a_1 = 20cm/s^2

Fig.VI.11.



v_{A1} = 2,5m/s, a_{A1} = 4m/s^2,
v_1 = 5m/s, a_1 = 3m/s^2

Fig.VI.12.

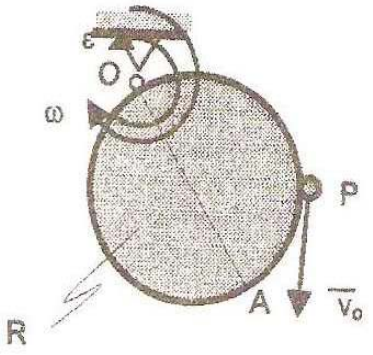


Fig.VI.13.

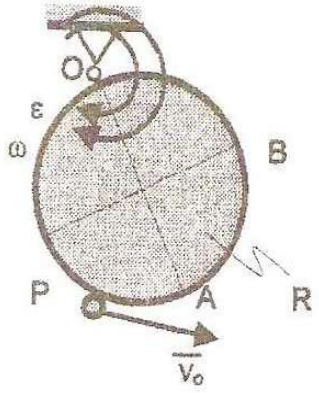


Fig.VI.14.

Problema VI.15.

Discul de rază R = 40 cm se rostogolește fără alunecare , în plan vertical, pe o dreaptă fixă orizontală având viteza centrului constantă v_o = 50 cm/s. În

același timp pe periferia discului se deplasează, uniform față de disc, cu viteza $v_P = 60$ cm/s un punct material P (Fig.VI.15.). La momentul t_1 punctul P se află în punctul A (extremitatea superioară a diametrului vertical din acel moment) . Să se calculeze și reprezinte viteza absolută și accelerația absolută a punctului P la momentul t_1.

Problema VI.16.

Discul de rază R = 0,35 m se rostogolește fără alunecare , în plan vertical, pe o dreaptă fixă orizontală având viteza centrului constantă $v_o = 0,85$ m/s. În același timp pe periferia discului se deplasează, uniform față de disc, cu viteza $v_P = 1,2$ m/s un punct material P (Fig.VI.16.). La momentul t_1 punctul P se află în punctul A (extremitatea din stânga a diametrului orizontal din acel moment) . Să se calculeze și reprezinte viteza absolută și accelerația absolută a punctului P la momentul t_1.

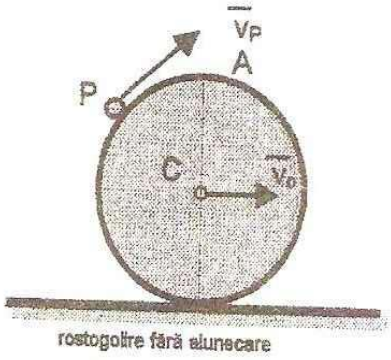


Fig.VI.15.

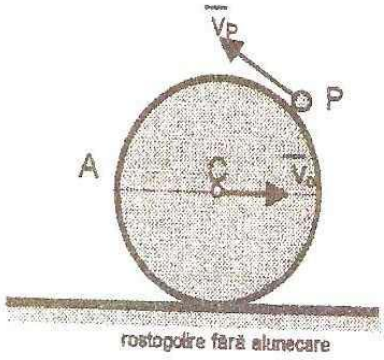


Fig.VI.16.

Problema VI.17.

Discul de rază R = 0,25 m se rostogolește fără alunecare , în plan vertical, pe o dreaptă fixă orizontală având viteza centrului constantă $v_o = 0,45$ m/s. În același timp pe diametrul AB al discului alunecă, cu viteza $\overline{v}_P$ și accelerația $\overline{a}_P$, față de disc, un punct material P (Fig.VI.17.). La momentul t_1 diametrul AB devine vertical, punctul P se află în punctul A (extremitatea superioară a diametrului AB în acel moment), viteza $\overline{v}_P$ are mărimea $v_{P1} = 0,6$ m/s, accelerația $\overline{a}_P$ are mărimea $a_{P1} = 0,5 m/s^2$ și cu sensurile din figură. Să se calculeze și reprezinte viteza absolută și accelerația absolută a punctului P la momentul t_1.

Problema VI.18.

Discul de rază R = 50 cm se rostogolește fără alunecare , în plan vertical, pe o dreaptă fixă orizontală având viteza centrului constantă $v_o = 15$ cm/s. În același timp pe diametrul AB al discului alunecă, cu viteza $\overline{v}_P$ și accelerația $\overline{a}_P$, față de disc, un punct material P (Fig.VI.18.). La momentul t_1 diametrul AB devine orizontal, punctul P se află în punctul A (extremitatea din dreapta a diametrului AB în acel moment), viteza $\overline{v}_P$ are mărimea $v_{P1} = 45$ cm/s, accelerația $\overline{a}_P$ are mărimea $a_{P1} = 60$

cm/s^2 şi cu sensurile din figură. Să se calculeze şi reprezinte viteza absolută şi acceleraţia absolută a punctului P la momentul t_1.

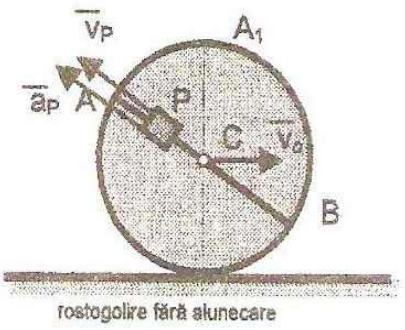


Fig.VI.17.

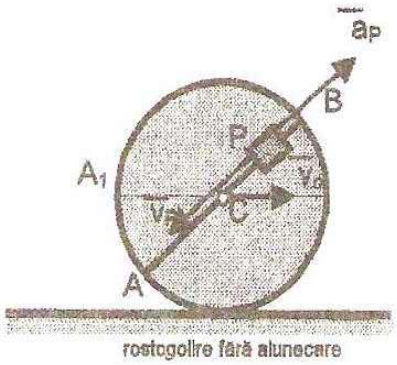


Fig.VI.18.

Problema VI.19.

Discul de rază R = 0,2m se roteşte în plan vertical în jurul punctului fix O cu viteza unghiulară ω şi acceleraţia unghiulară ε. În acelaşi timp pe coarda AB (OA şi BCD sunt două diametre perpendiculare) alunecă cu viteza $\bar{v}$ şi acceleraţia $\bar{a}$, faţă de disc, o culisă P. La momentul t_1 diametrul OA devine vertical, culisa P ajunge în A, vitezele şi acceleraţiile: ω, ε, $\bar{v}$, $\bar{a}$ vor avea mărimile : ω_1 = 2rad/s, ε_1 = 5rad/s^2, v_1 = 0,4m/s respectiv a_1 = 0,6m/s^2 şi cu sensurile reprezentate în figura VI.19. Să se calculeze şi reprezinte viteza absolută şi acceleraţia absolută a culisei la momentul t_1.

Problema VI.20.

Discul de rază R = 0,4m se roteşte în plan vertical în jurul punctului fix O cu viteza unghiulară ω şi acceleraţia unghiulară ε. În acelaşi timp pe coarda AB (OA şi BCD sunt două diametre perpendiculare) alunecă cu viteza $\bar{v}$ şi acceleraţia $\bar{a}$, faţă de disc, o culisă P. La momentul t_1 diametrul OA devine vertical, culisa P ajunge în B, vitezele şi acceleraţiile: ω, ε, $\bar{v}$, $\bar{a}$ vor avea mărimile : ω_1 = 3rad/s, ε_1 = 5rad/s^2, v_1 = 0,5m/s respectiv a_1 = 0,6m/s^2 şi cu sensurile reprezentate în figura VI.20. Să se calculeze şi reprezinte viteza absolută şi acceleraţia absolută a culisei la momentul t_1.

Problema VI.21.

Bara AB de lungime l_{AB} = 2 m se mişcă în plan vertical descriind cu cele două extremităţi două drepte fixe perpendiculare (una orizontală şi una verticală). Viteza extremităţii A este constantă v_A = 1,5 m/s. În acelaşi timp pe bară alunecă cu viteza $\bar{v}$ şi acceleraţia $\bar{a}$, faţă de bară, o culisă P. La momentul t_1 al mişcării sistemului bara face cu orizontala un unghi de 45°, culisa ajunge în A, viteza $\bar{v}$ şi acceleraţia $\bar{a}$ având valorile: v_1 = 2 m/s şi a_1 = 3 m/s^2, cu sensurile reprezentate în figura VI.21. Să se calculeze şi reprezinte viteza absolută şi acceleraţia absolută a culisei în momentul t_1.

Problema VI.22.

Triunghiul echilateral OAB de latură l = 60 cm efectuează , în plan vertical , o rotație în jurul vârfului O cu viteza unghiulară ω și accelerația unghiulară ε. În același timp pe latura AB alunecă o culisă cu viteza $\overline{v}$ și accelerația $\overline{a}$, în raport cu triunghiul. La momentul t_1 al mișcării latura AB devine orizontală, culisa P ajunge în B iar vitezele și accelerațiile vor avea valorile: ω_1 = 1 rad/s, ε_1 = 2 rad/s^2 , v_1 = 80 cm/s respectiv a_1 = 100 cm/s^2 cu sensurile din figura VI.22. Să se calculeze și reprezinte viteza și accelerația absolută a culisei la momentul indicat.

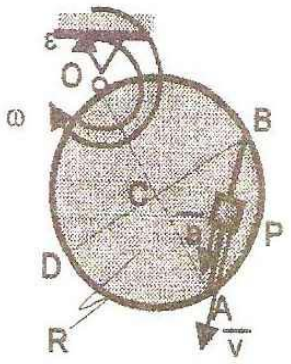


Fig.VI.19.

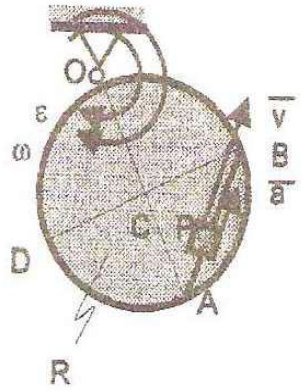


Fig.VI.20.

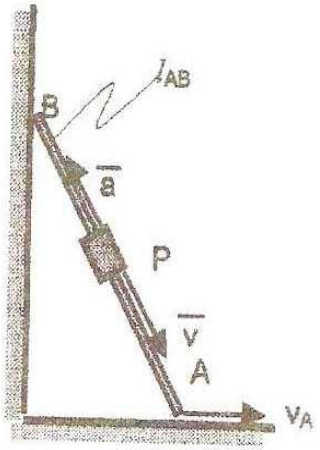


Fig.VI.21.

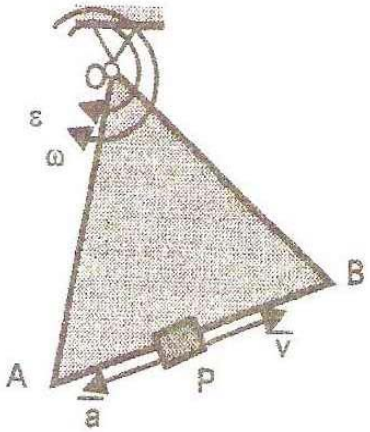


Fig.VI.22.

CAPITOLUL VII. PRINCIPIUL LUCRULUI MECANIC VIRTUAL PENTRU STAREA DE REPAUS

VII.1. CALCULUL REACȚIUNILOR LA GRINZILE GERBER

Problemele VII.1.1. – VII.1.4

Folosind principiul lucrului mecanic virtual să se calculeze reacțiunile: V_A, M_A și V_B pentru grinda Gerber din figura VII.1.1. – VII.1.4.

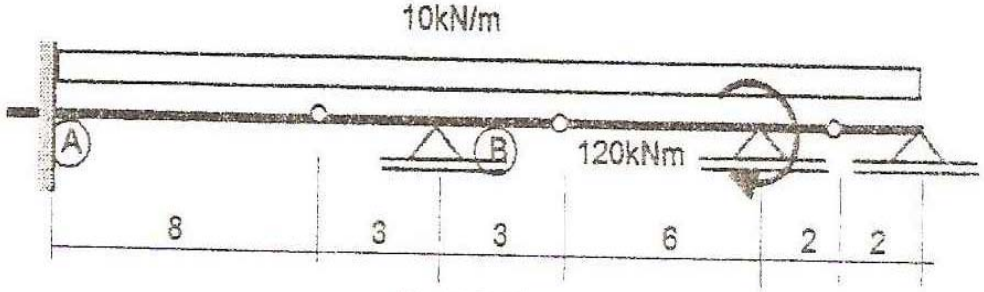


Fig.VII.1.1

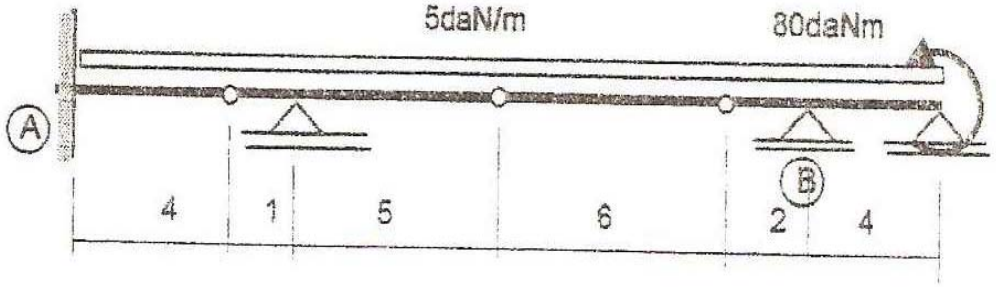


Fig.VII.1.2.

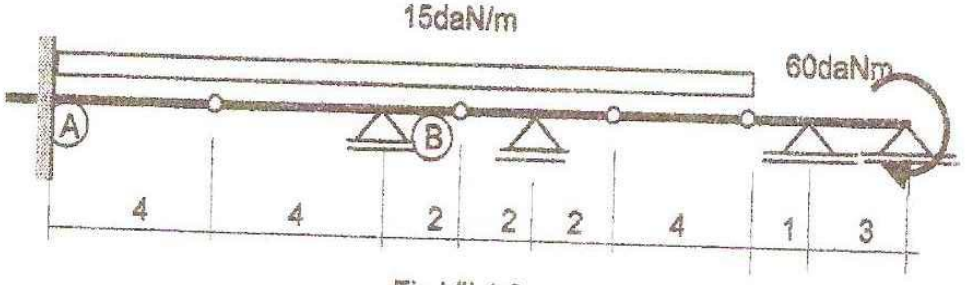


Fig.VII.1.3

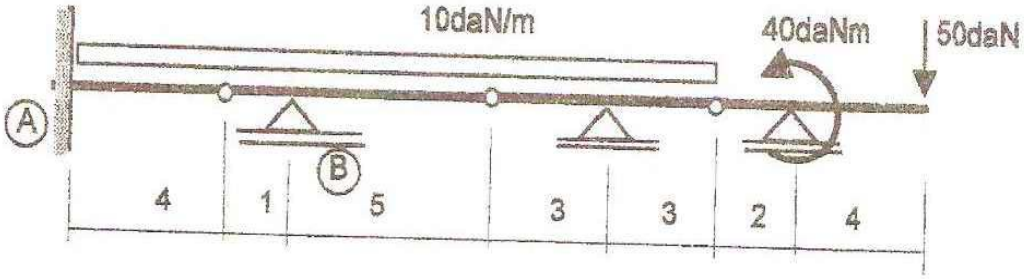


Fig.VII.1.4.

Problema VII.1.5. –VII.1.8.

Folosind principiul lucrului mecanic virtual să se calculeze reacțiunile: V_A, M_B și V_B pentru grinda Gerber din figura VII.1.5. – VII.1.8.

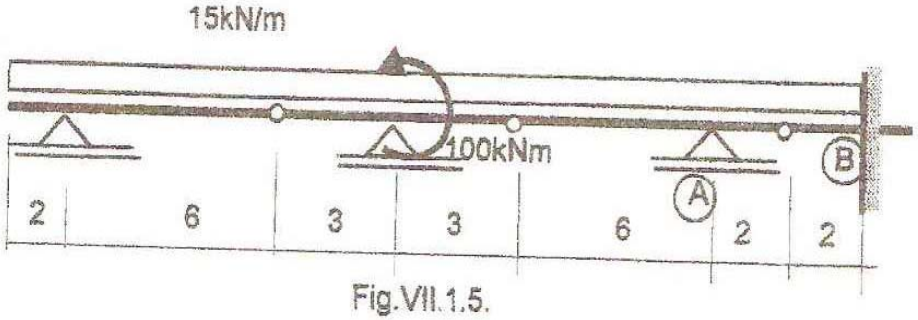


Fig.VII.1.5.

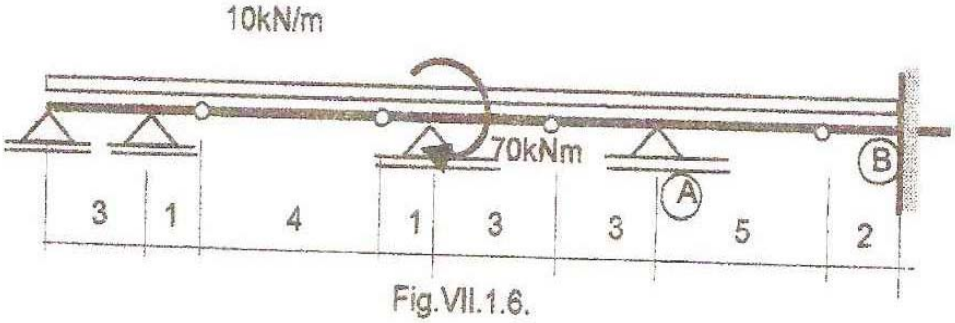


Fig.VII.1.6.

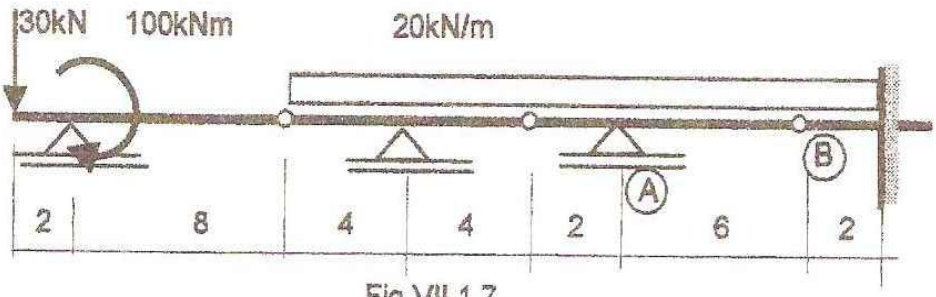


Fig.VII.1.7.

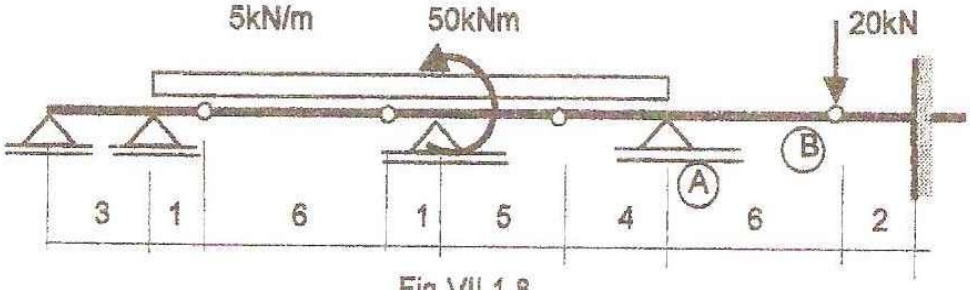


Fig.VII.1.8.

Problemele VII.1.9 – VII.20.

Folosind principiul lucrului mecanic virtual să se calculeze reacțiunile: V_A, V_B și V_C pentru grinda Gerber din figura VII.1.9. – VII.1.20.

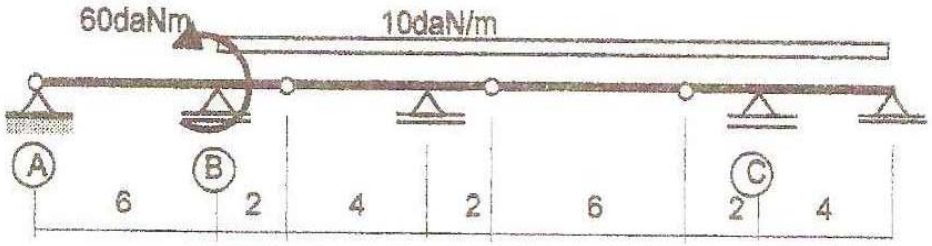


Fig.VII.1.9.

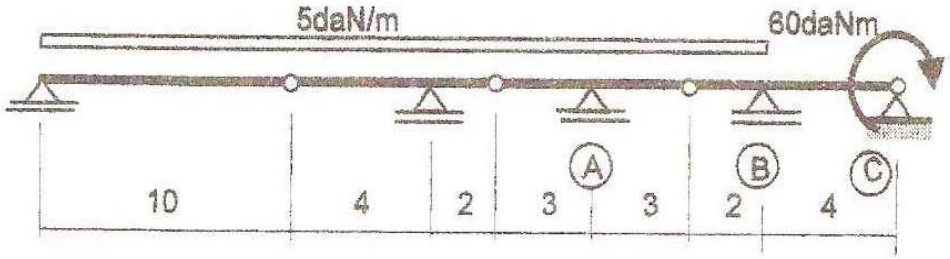


Fig.VII.1.10.

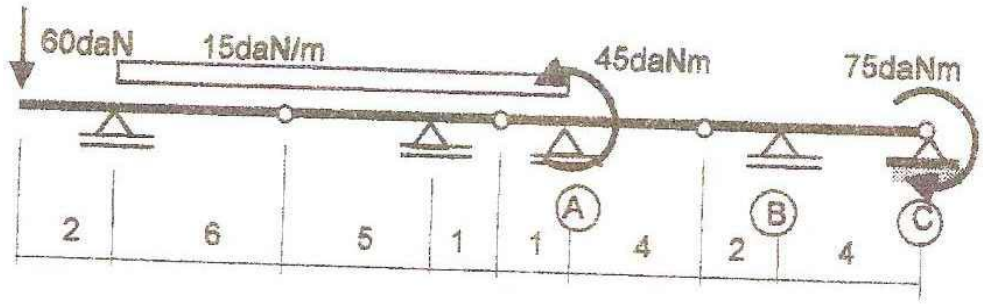


Fig.VII.1.11.

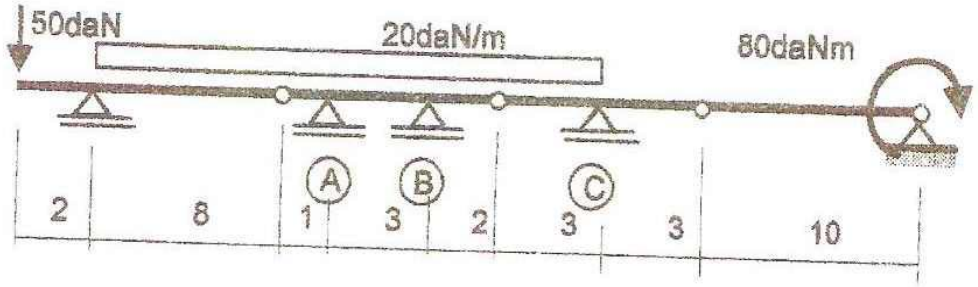


Fig.VII.1.12.

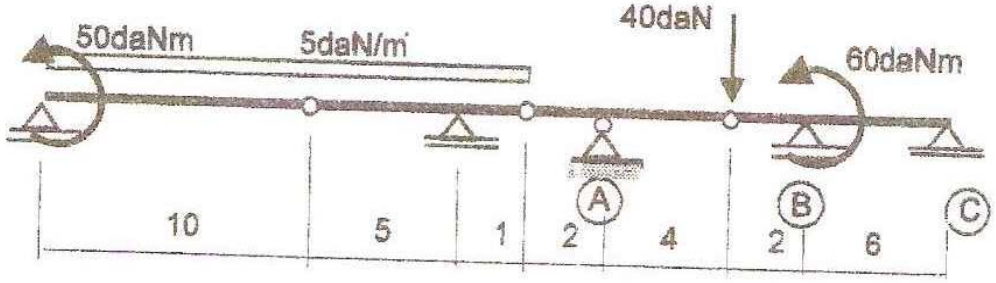


Fig.VII.1.13.

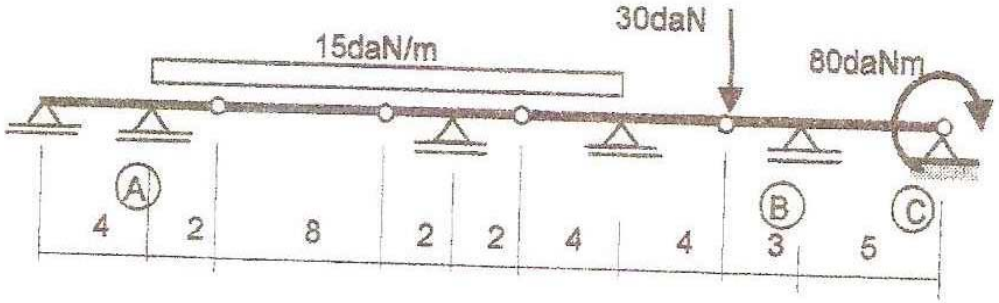


Fig.VII.14.

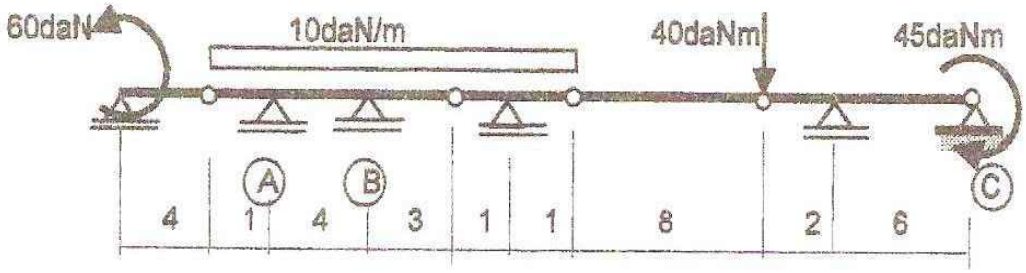


Fig.VII.1.15.

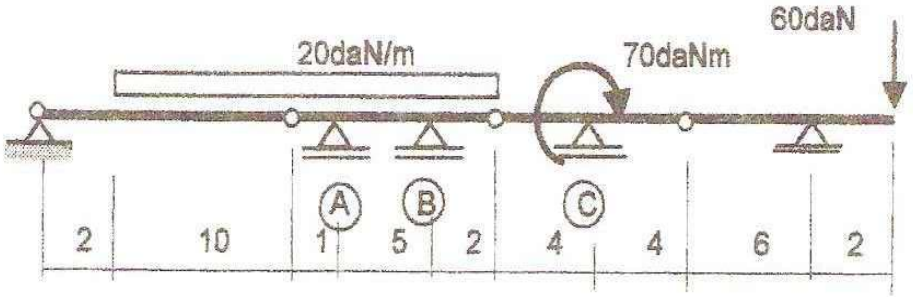


Fig.VII.1.16.

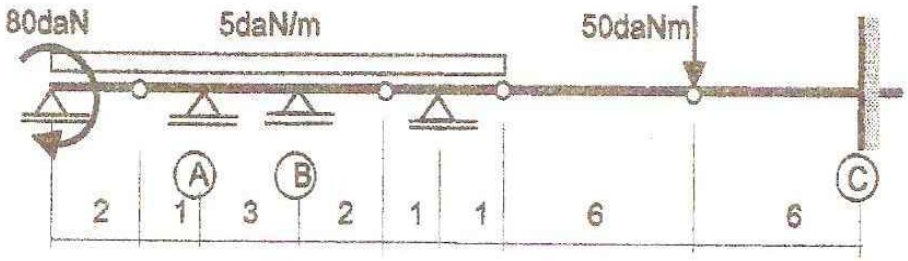


Fig.VII.1.17.

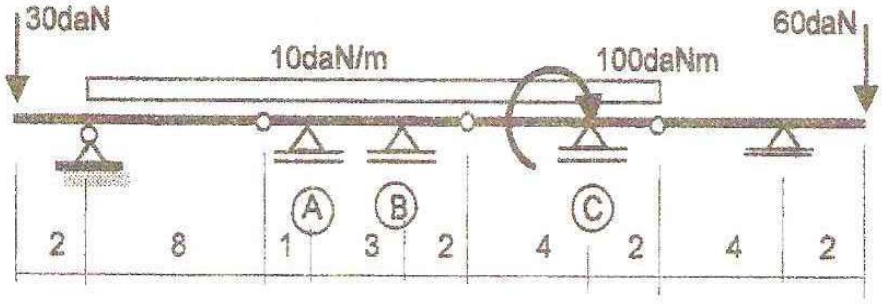


Fig.VII.1.18.

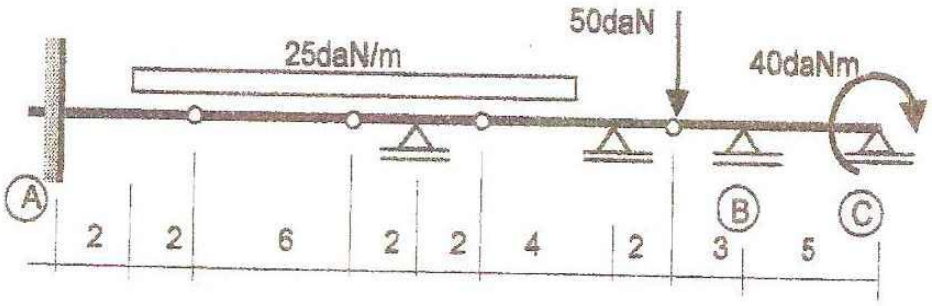


Fig.VII.19.

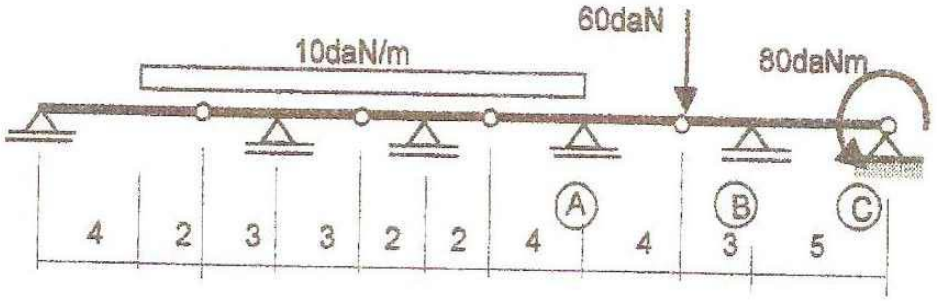


Fig.VII.20.

VII.2. CALCULUL REACȚIUNILOR LA CADRE

Problemele VII.2.1. – VII.2.10.

Folosind principiul lucrului mecanic virtual să se calculeze reacțiunile H_A, V_A și M_A din încastrarea A pentru cadrul reprezentat în figura VII.2.1. – VII.2.10 .

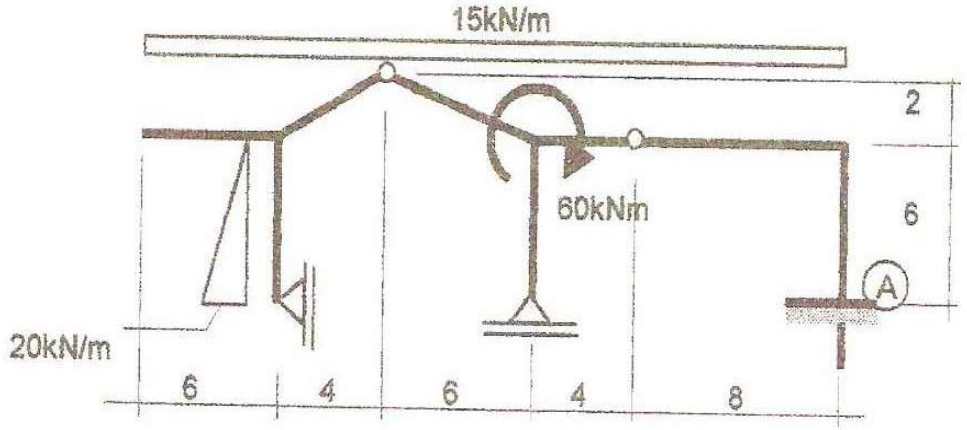


Fig.VII.2.1.

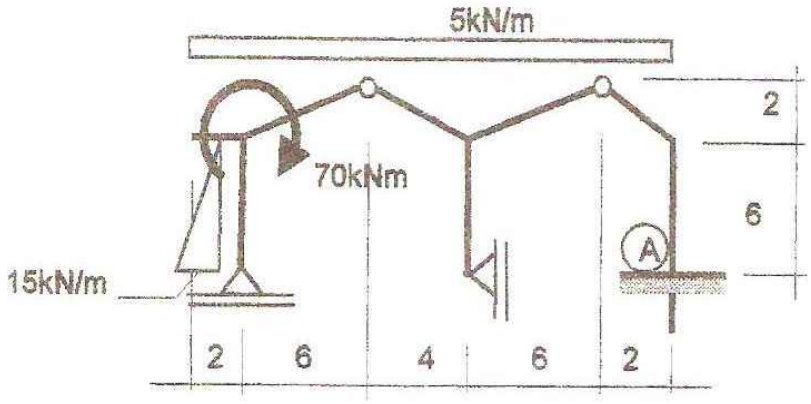


Fig.VII.2.2.

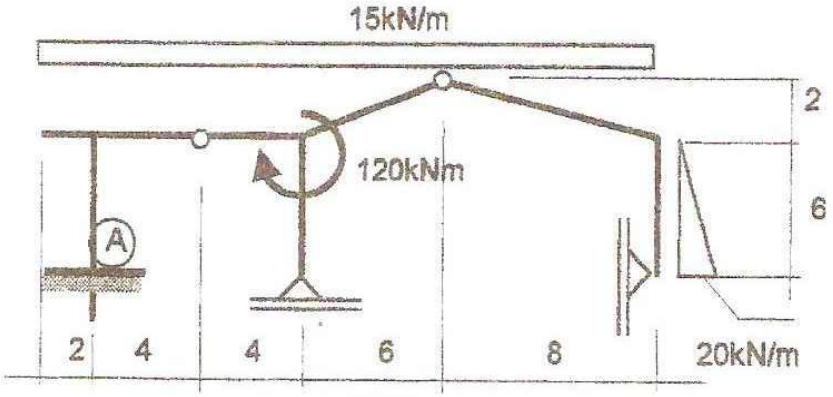


Fig.VII.2.3.

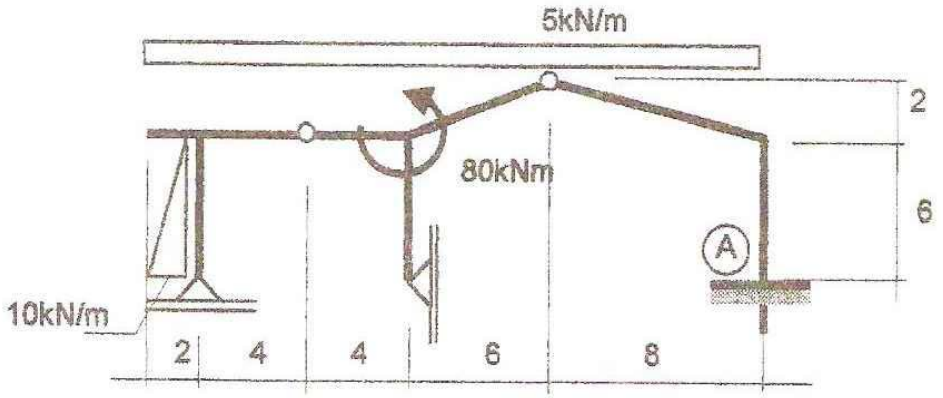


Fig.VII.2.4.

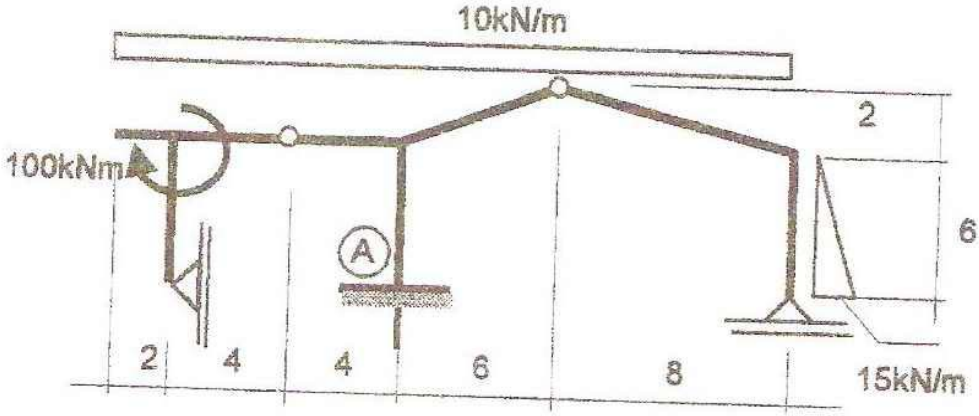


Fig.VII.2.5.

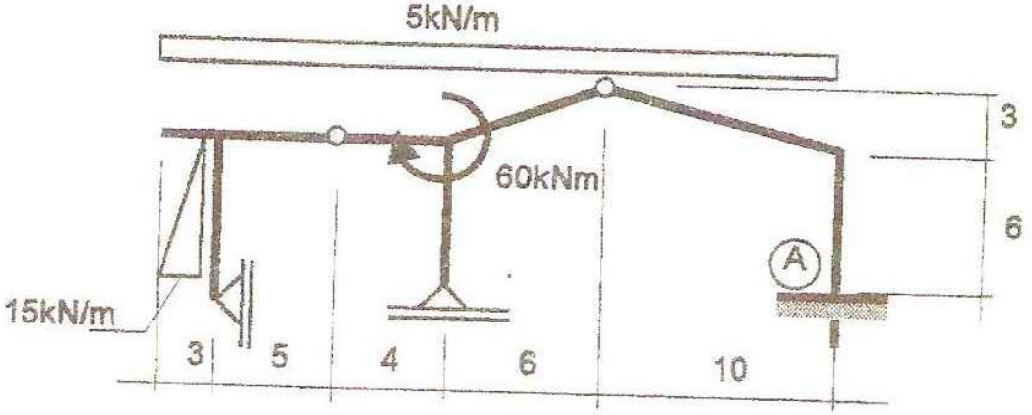


Fig.VII.2.6.

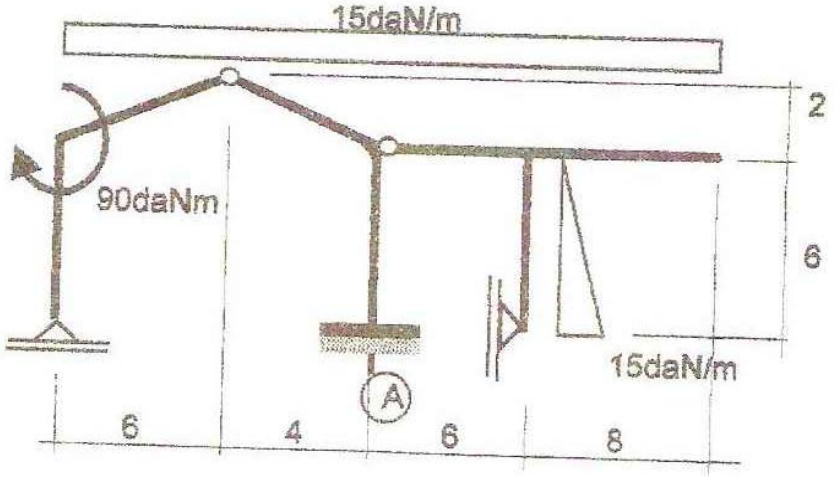


Fig. VII.2.7.

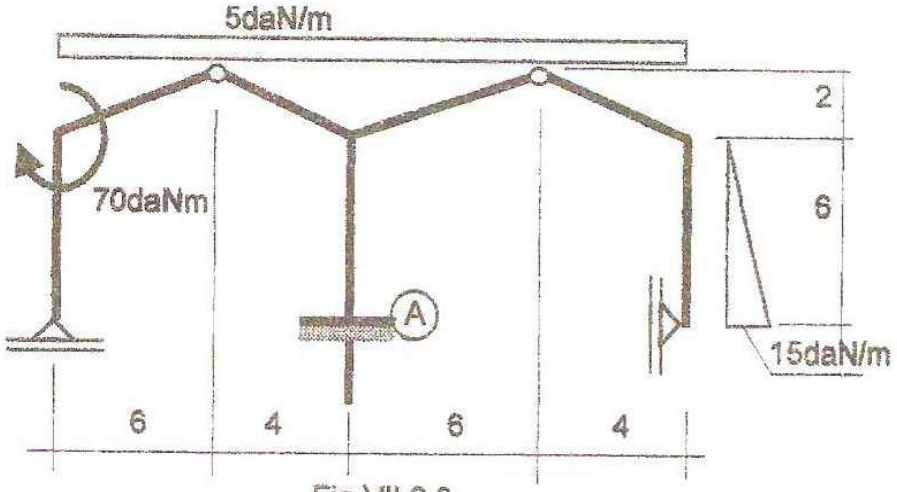


Fig.VII.2.8.

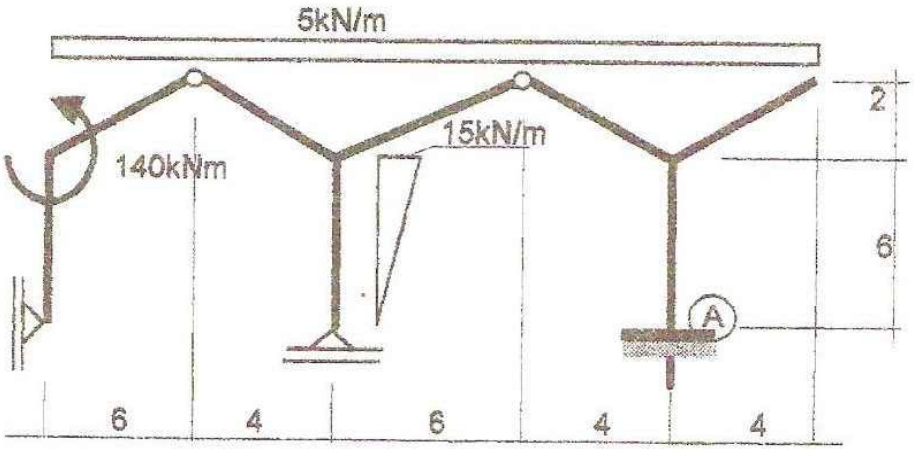


Fig. VII.2.9.

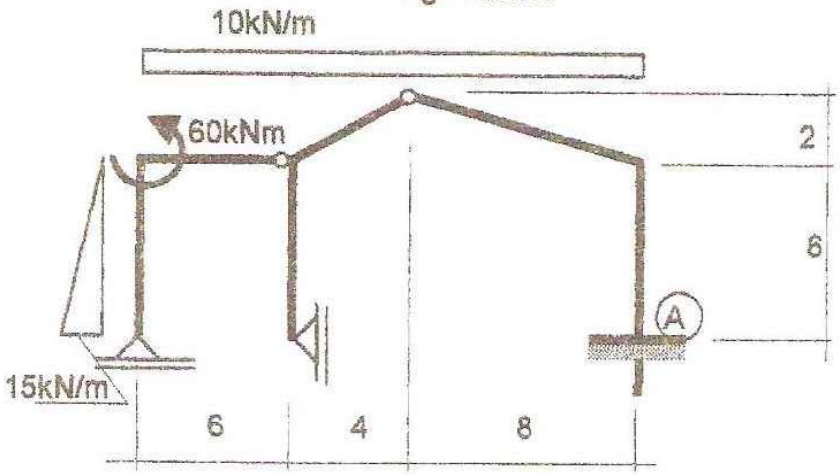


Fig.VII.2.10.

Problemele VII.2.11. – VII.2.20.

Folosind principiul lucrului mecanic virtual să se calculeze reacţiunile V_A, H_A şi V_B pentru cadrul reprezentat în figura VII.2.11.-VII.2.20.

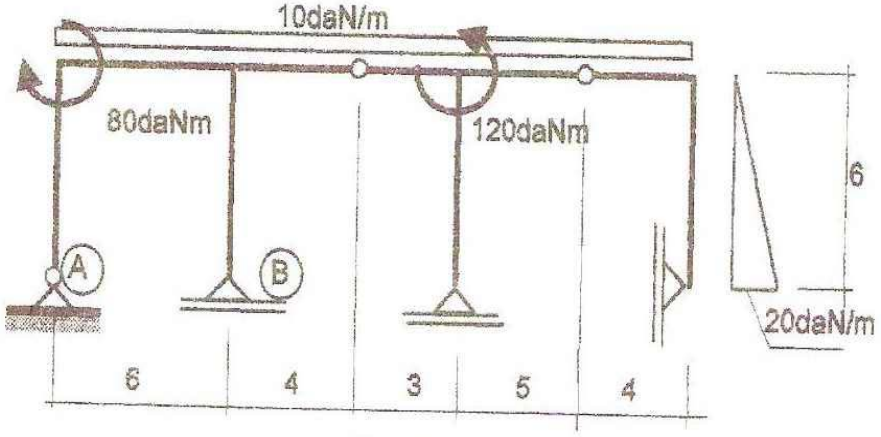


Fig.VII.2.11.

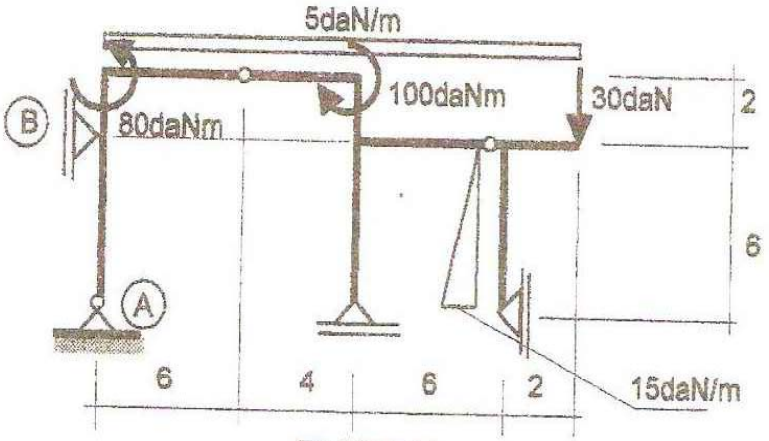


Fig.VII.2.12.

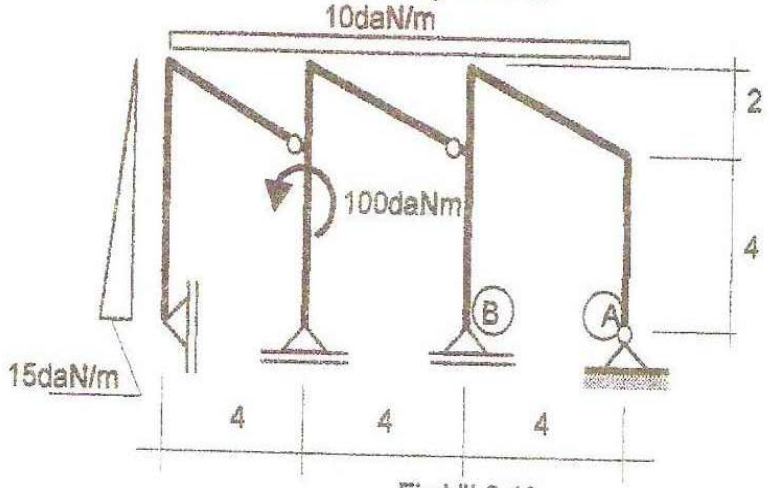


Fig.VII.2.13.

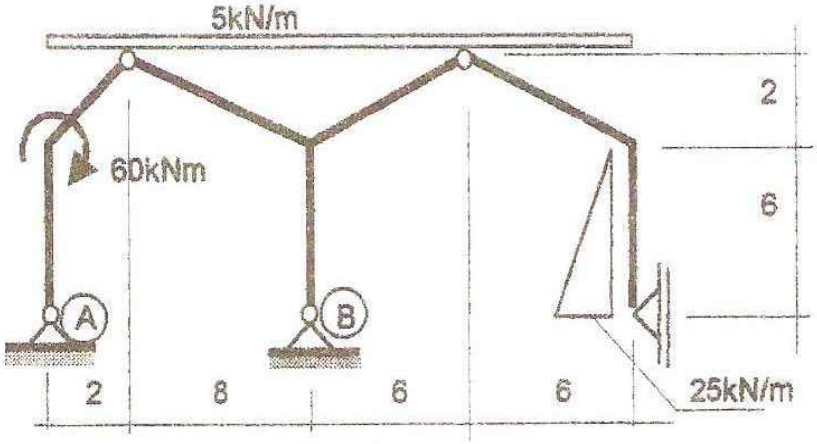


Fig.VII.2.14.

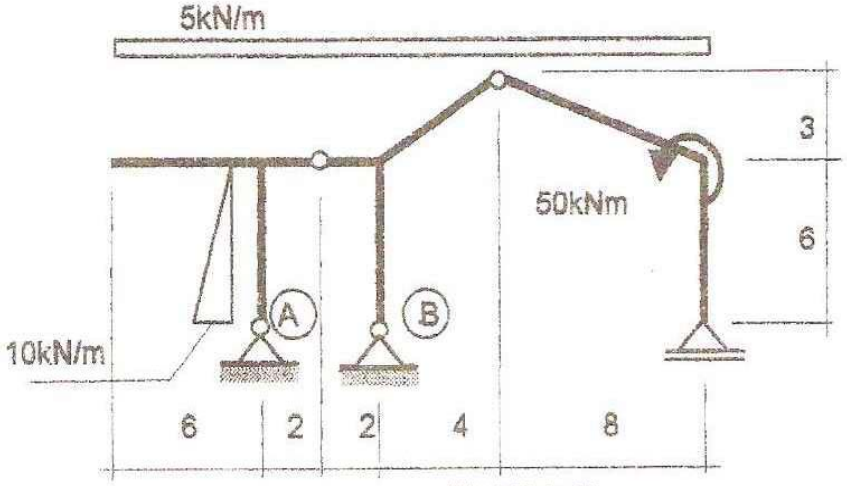


Fig.VII.2.15.

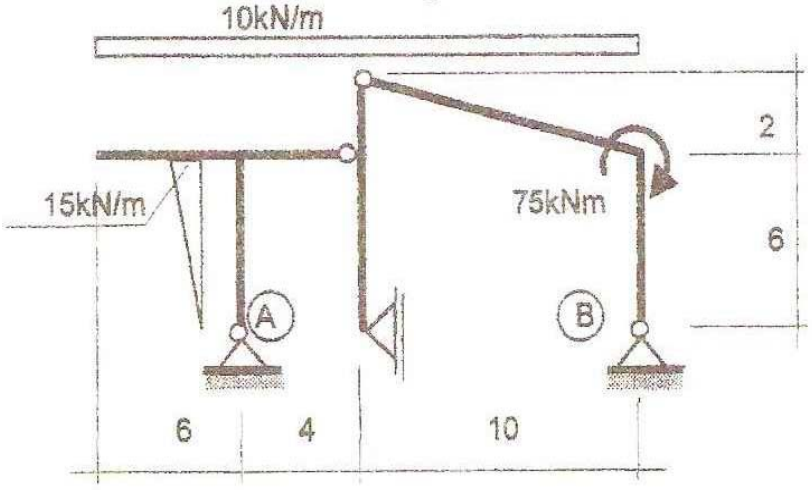


Fig.VII.2.16.

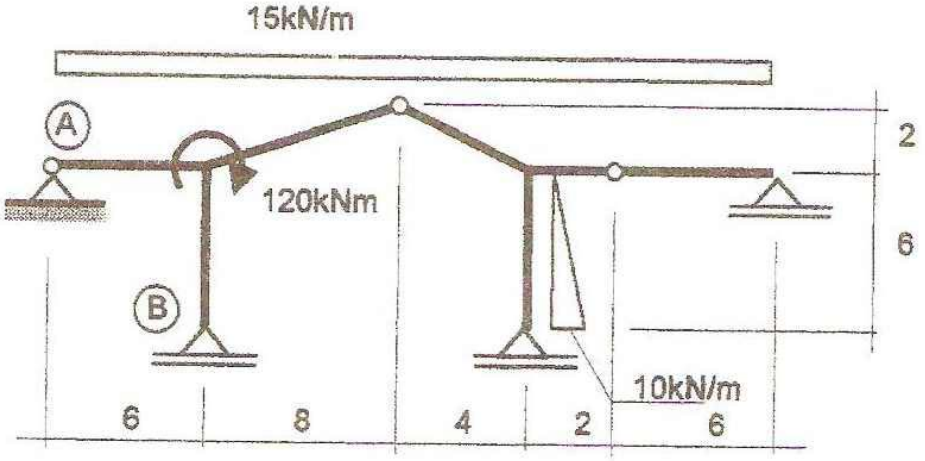


Fig.VII.2.17.

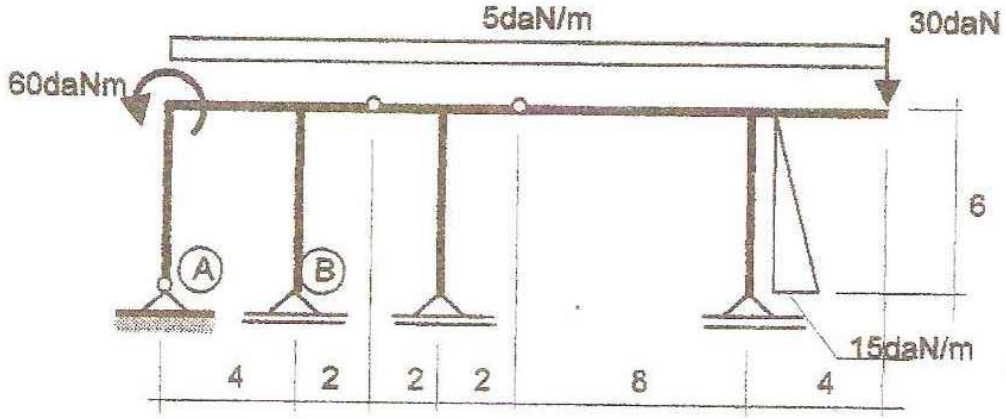


Fig.VII.2.18.

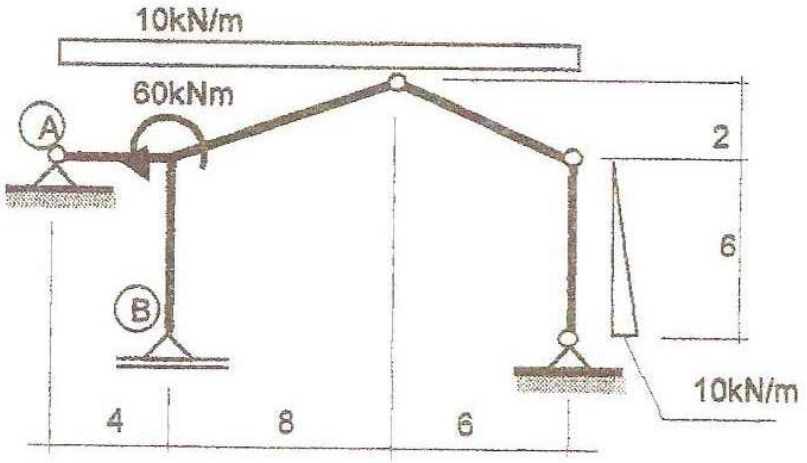


Fig.VII.2.19.

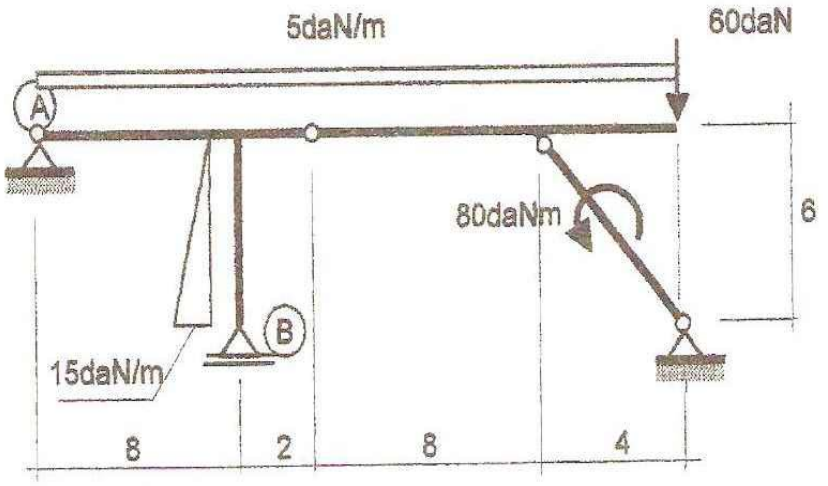


Fig.VII.2.20.

VII.3. CALCULUL EFORTURILOR ÎN BARELE GRINZILOR CU ZĂBRELE

Problemele VII.3.1. – VII.3.20.

Folosind principiul lucrului mecanic virtual să se calculeze eforturile în barele însemnate pentru grinzile reprezentate în figurile VII.3.1. – VII.3.20.

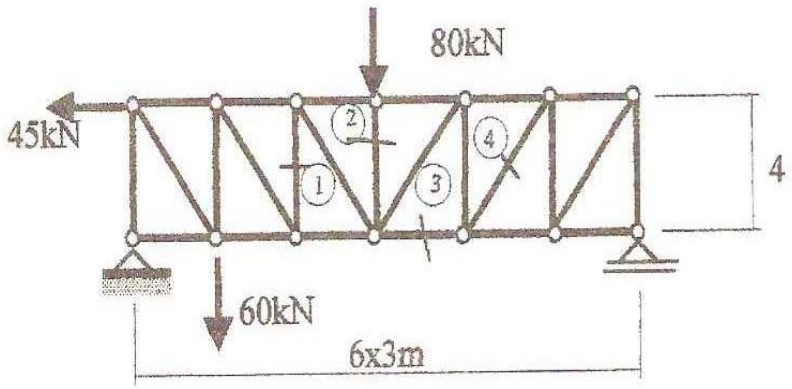


Fig. VII.3.1.

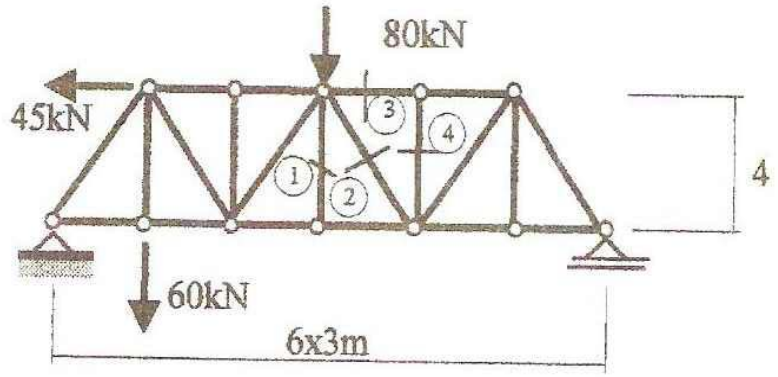


Fig. VII.3.2.

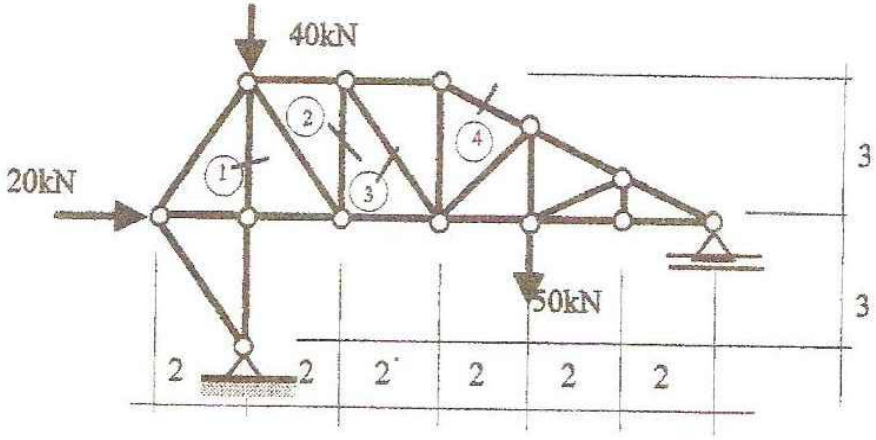


Fig. VII.3.3.

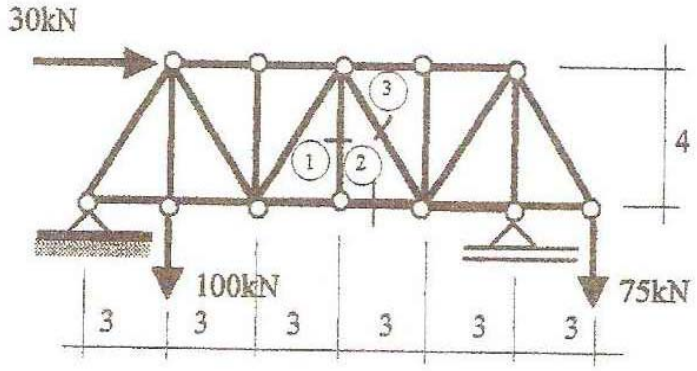


Fig. VII.3.4.

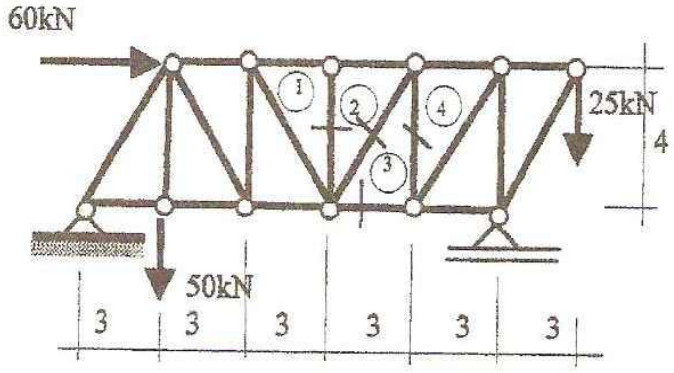


Fig. VII.3.5.

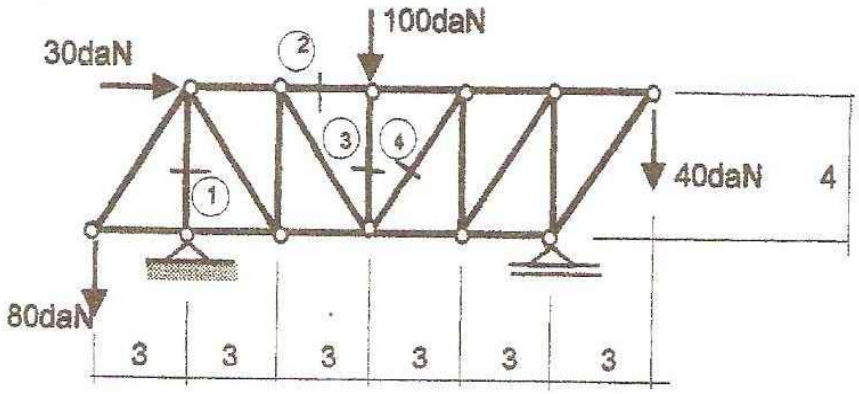


Fig.VII.3.6.

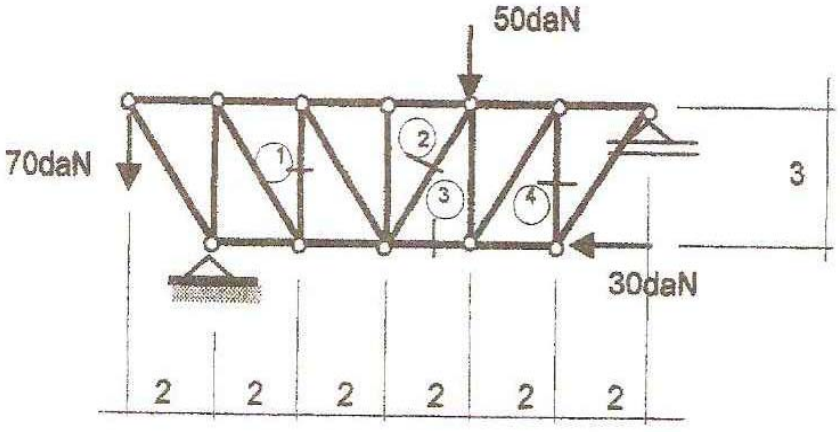


Fig.VII.3.7.

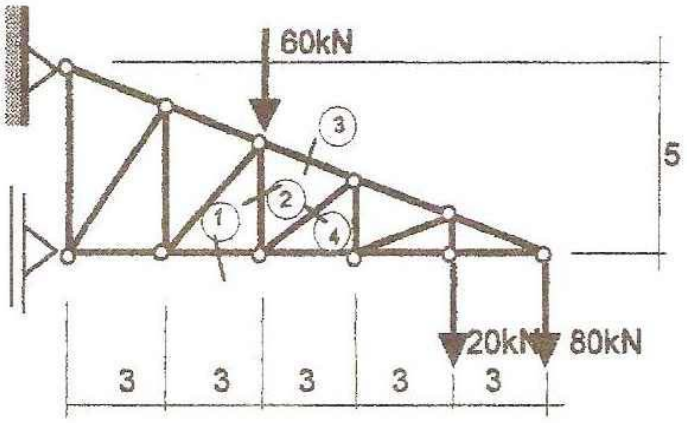


Fig.VII.3.8.

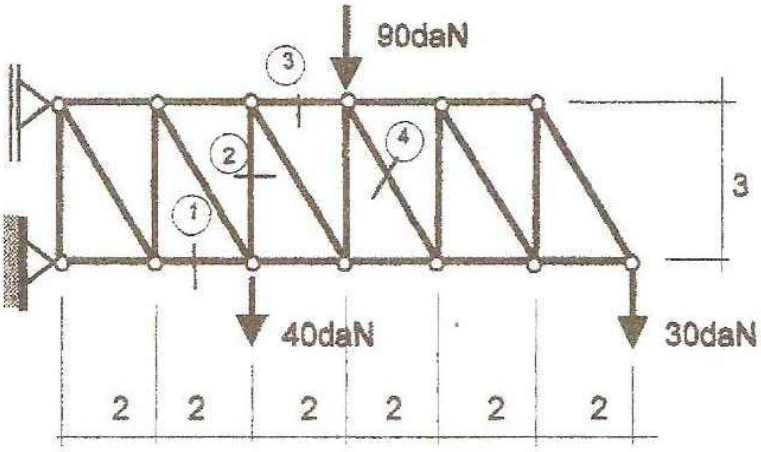


Fig.VII.3.9.

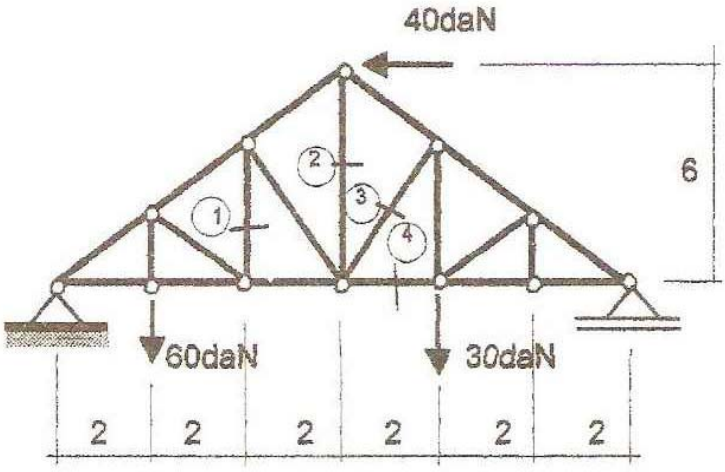


Fig.VII.3.10.

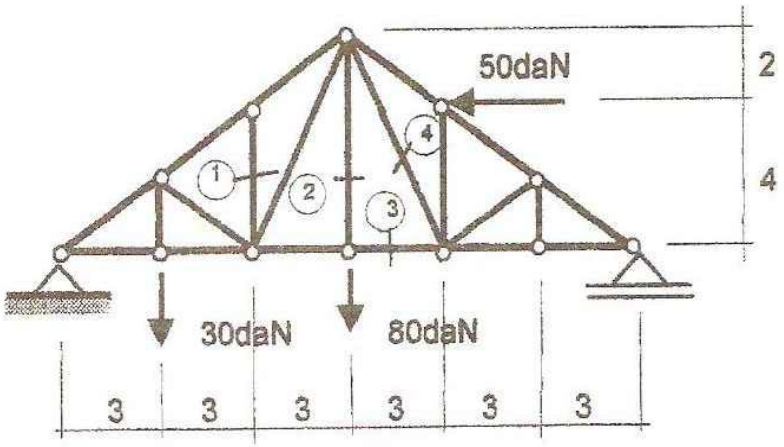


Fig.VII.3.11.

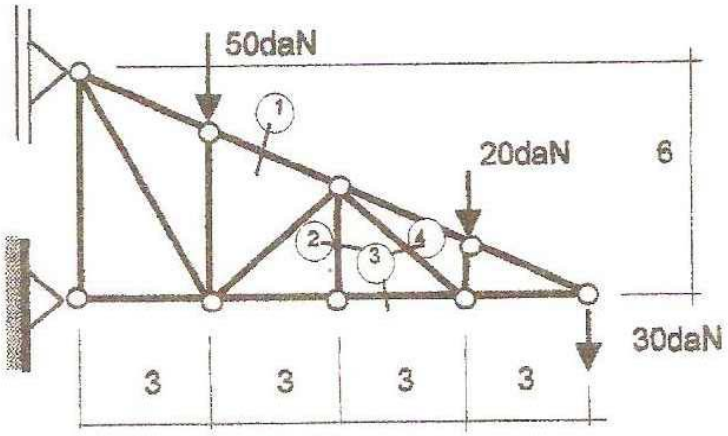


Fig.VII.3.12.

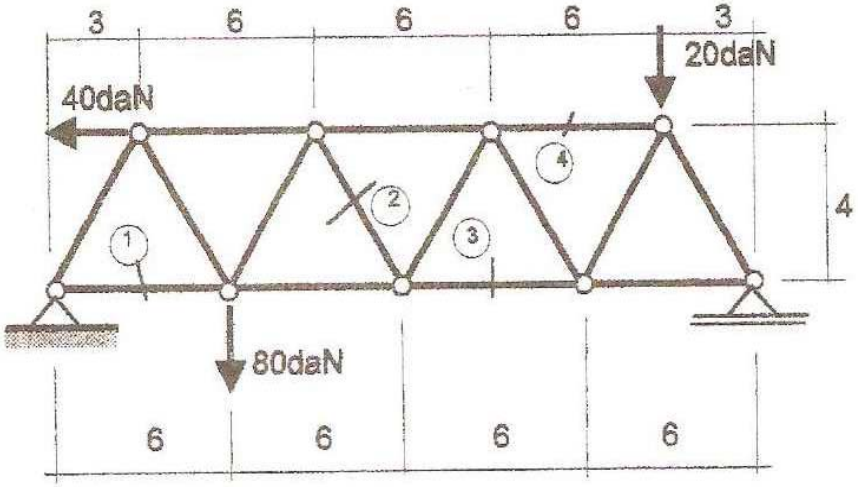


Fig.VII.3.13.

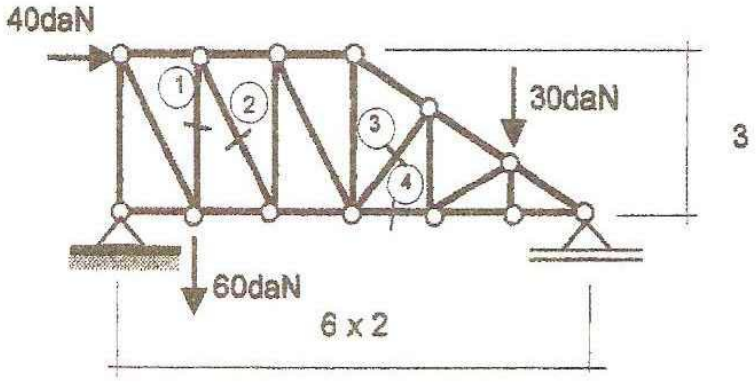


Fig.VII.3.14.

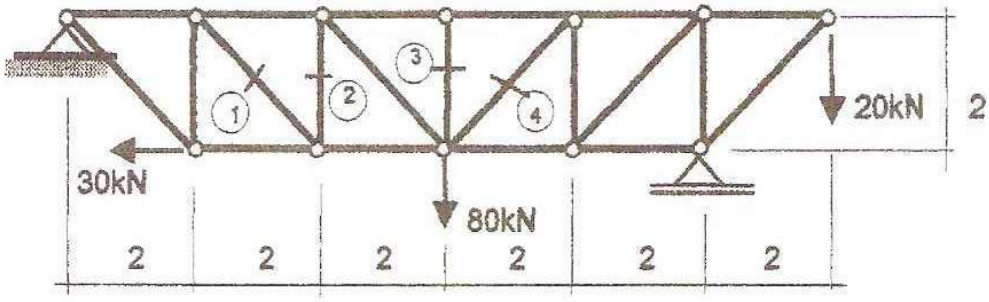


Fig.VII.3.15.

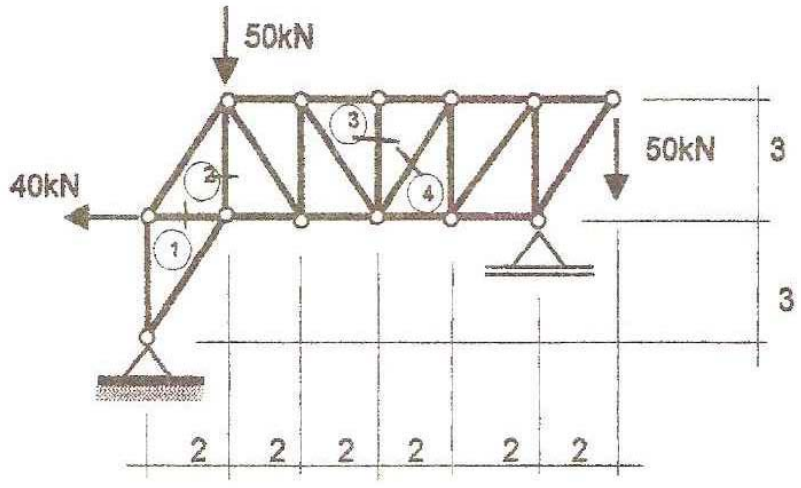


Fig.VII.3.16.

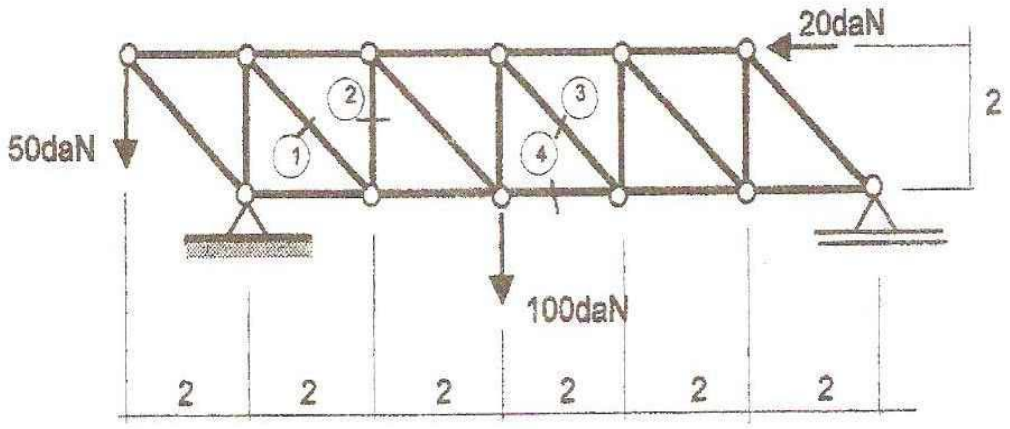


Fig.VII.3.17.

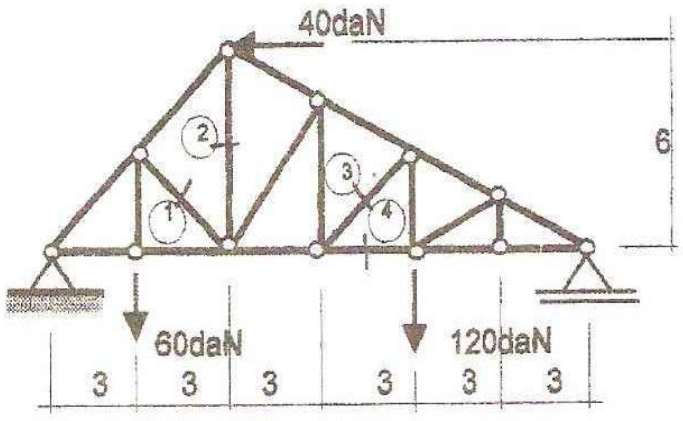
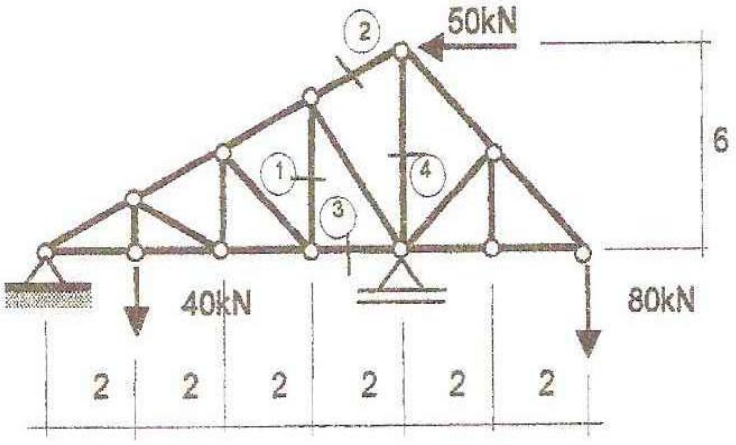


Fig.VII.3.18.



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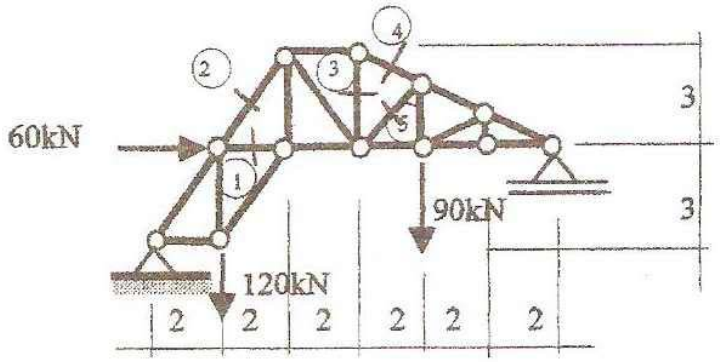


Fig. VII.3.20.

CAPITOLUL VIII. STUDIUL MIŞCĂRII UNUI SISTEM MECANIC

VIII.1. FOLOSIREA PRINCIPIULUI LUI D'ALEMBERT PENTRU STUDIUL MIŞCĂRII UNUI SISTEM DE SCRIPEŢI (METODA CINETO- STATICĂ)

Problemele VIII.1.1. – VIII.1.30.

Cunoscând faptul că în timpul mişcării sistemelor de scripeţi reprezentate în figurile VIII.1.1. – VIII.1.30. firele verticale rămân verticale (şi faptul că ele nu alunecă pe scripeţi) să se calculeze acceleraţiile punctelor materiale, ale centrelor scripeţilor mobili şi acceleraţiile unghiulare ale scripeţilor, precum şi tensiunile din fire.

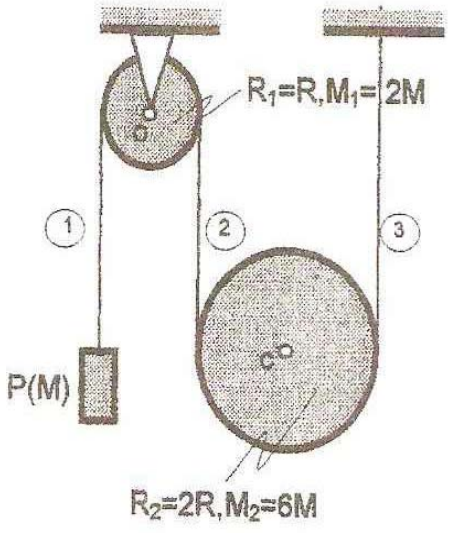


Fig.VIII.1.1.

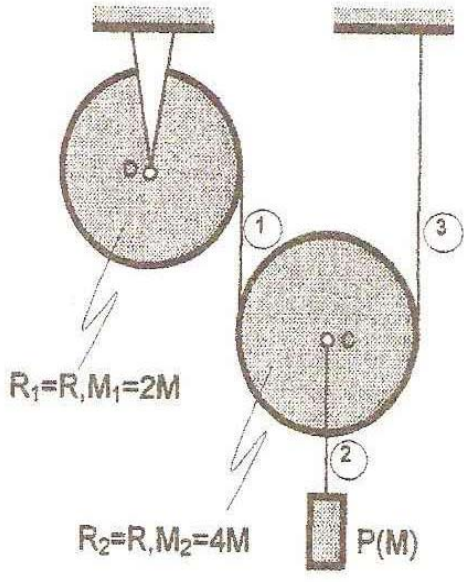


Fig.VIII.1.2.

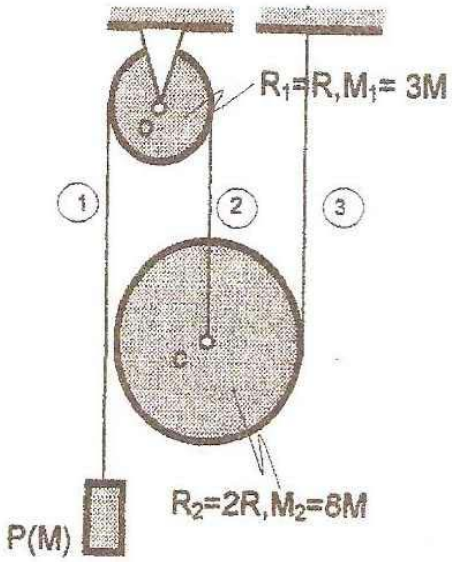


Fig.VIII.1.3.

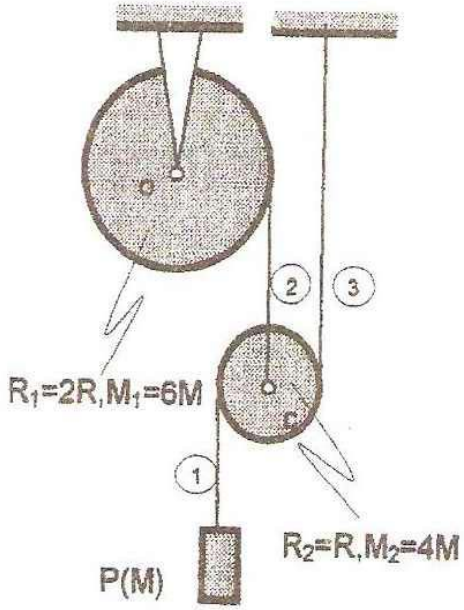


Fig.VIII.1.4.

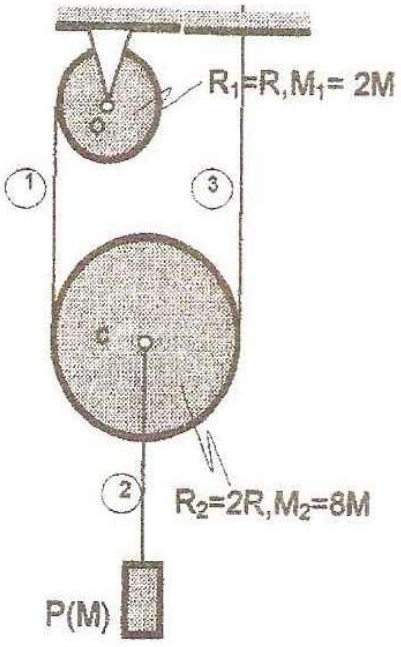


Fig.VIII.1.5.

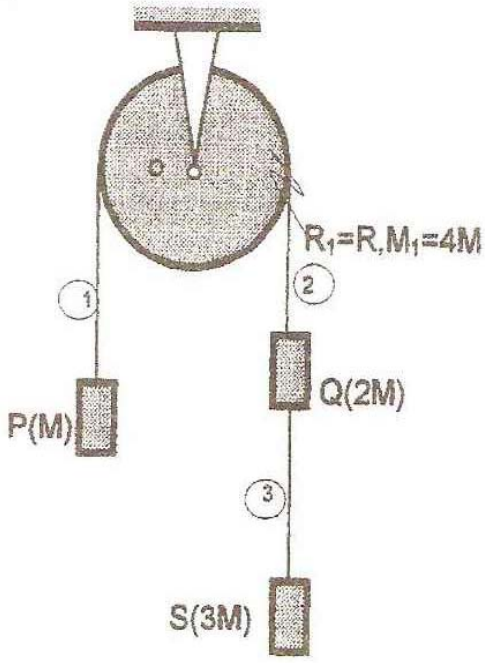


Fig.VIII.1.6.

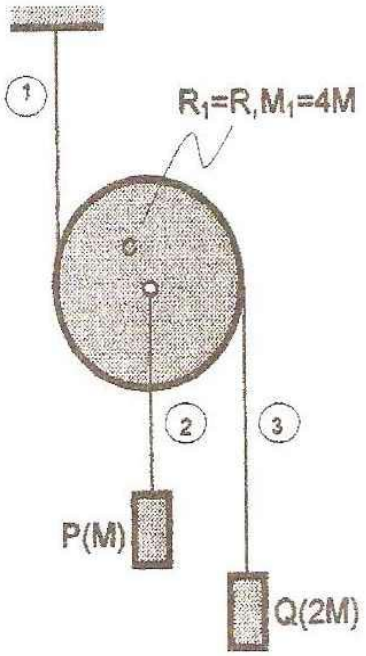


Fig.VIII.1.7.

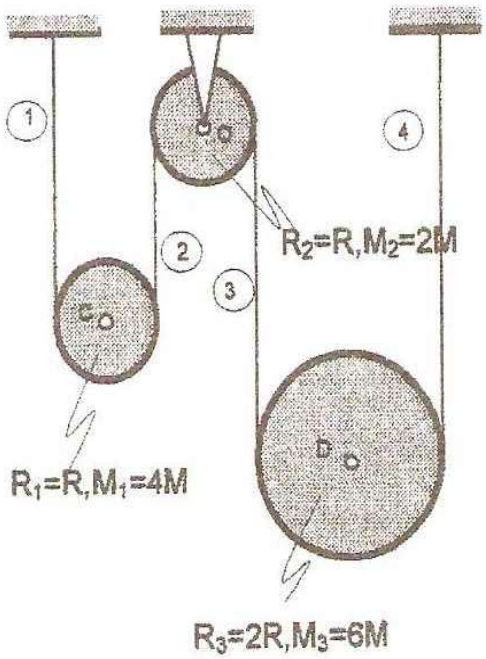


Fig.VIII.1.8.

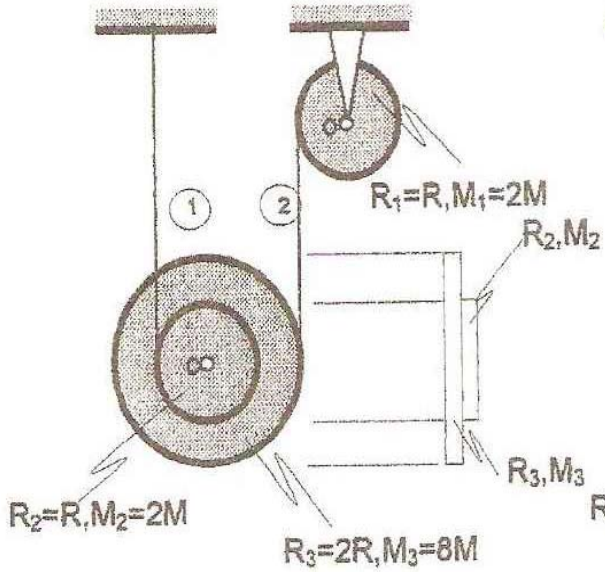


Fig.VIII.1.9.

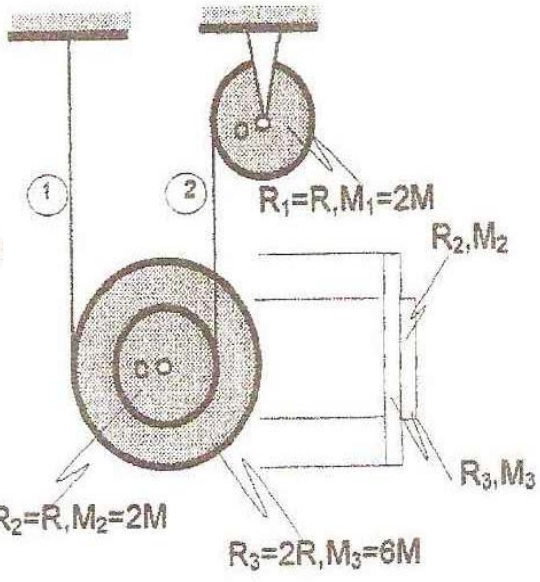


Fig.VIII.1.10.

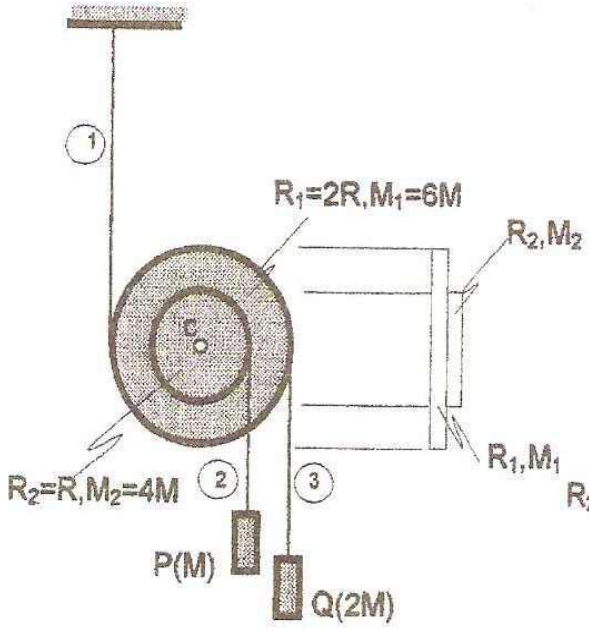


Fig.VIII.1.11.

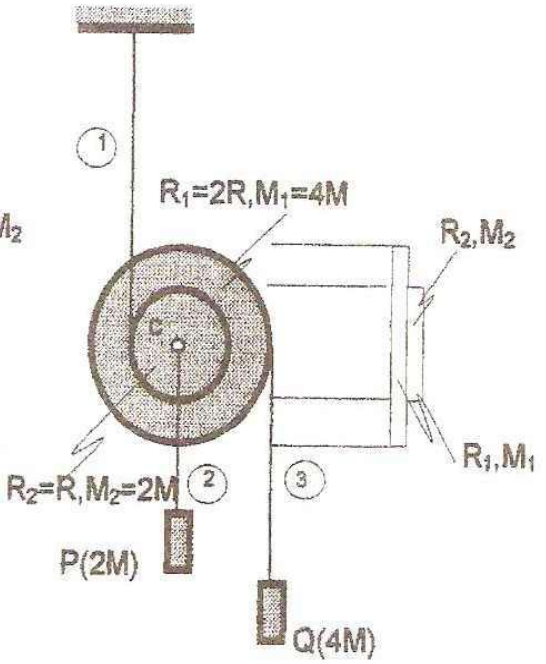


Fig.VIII.1.12.

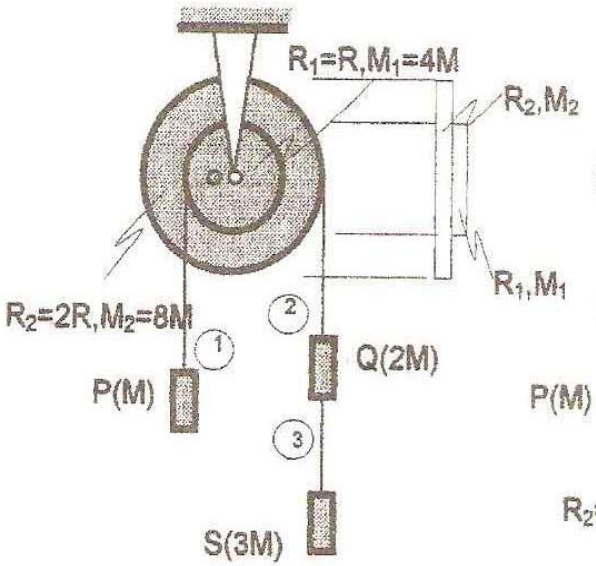


Fig.VIII.1.13.

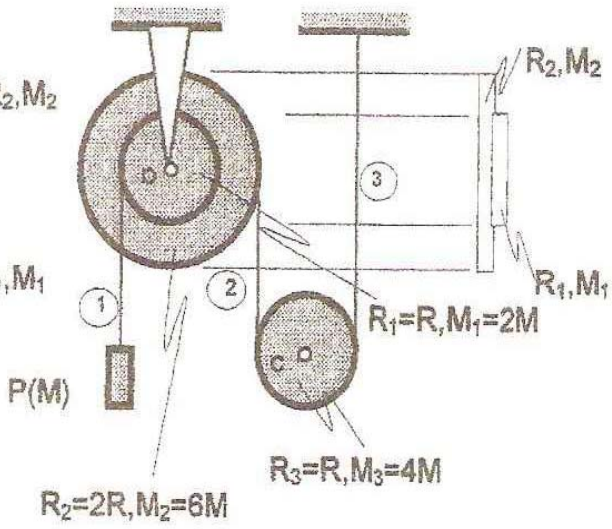


Fig.VIII.1.14

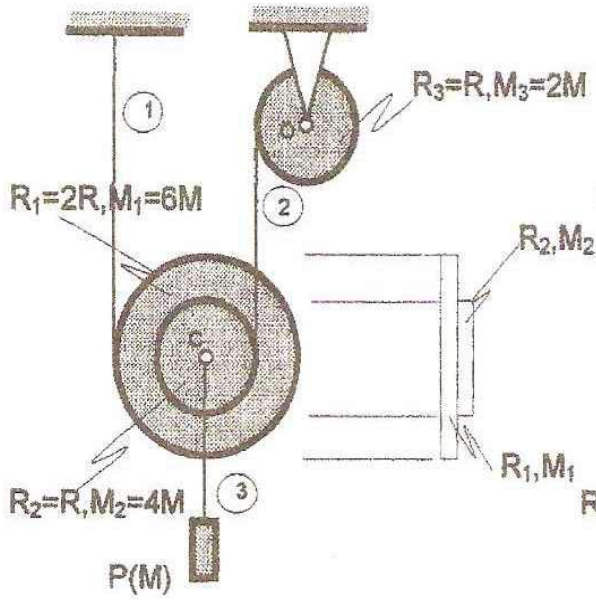


Fig.VIII.1.15.

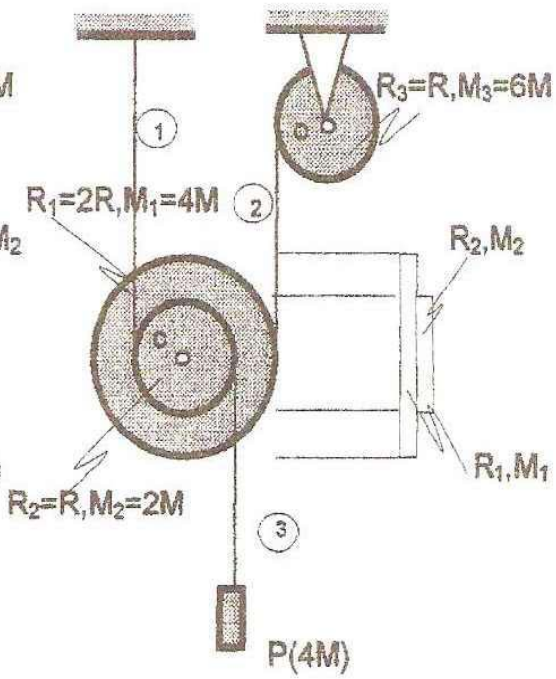


Fig.VIII.1.16.

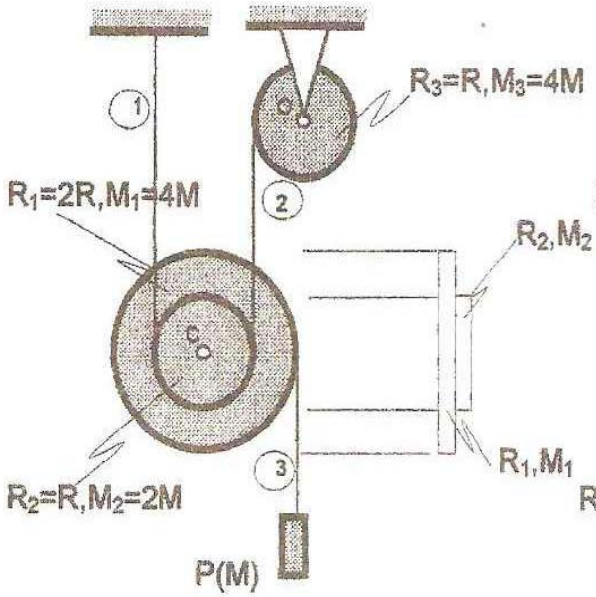


Fig.VIII.1.17.

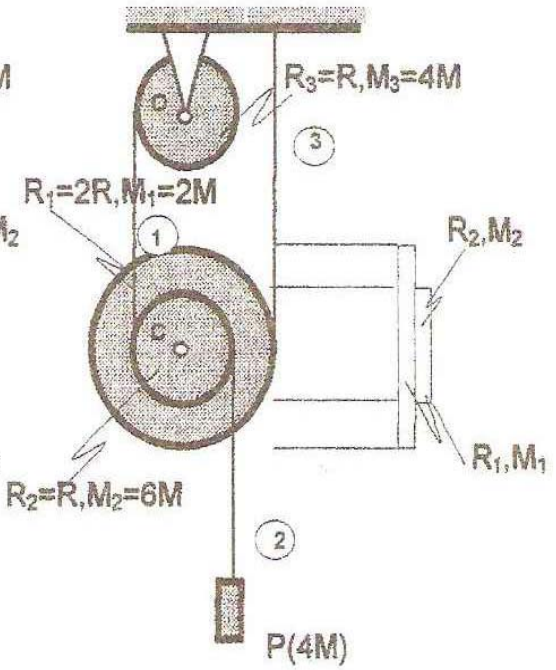
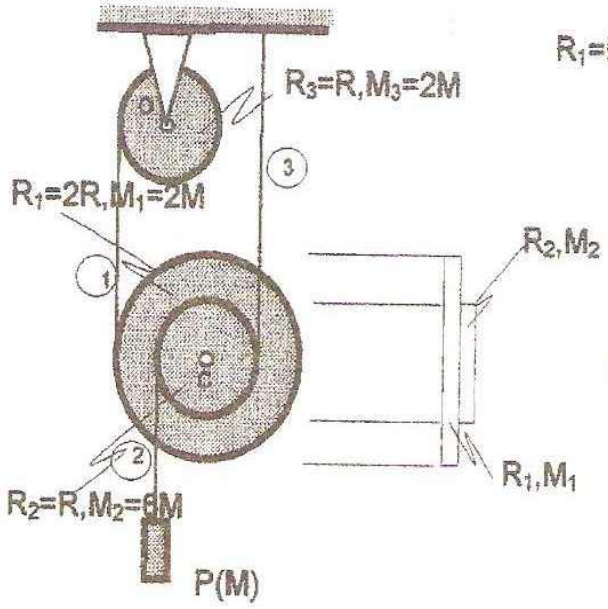


Fig.VIII.1.18.



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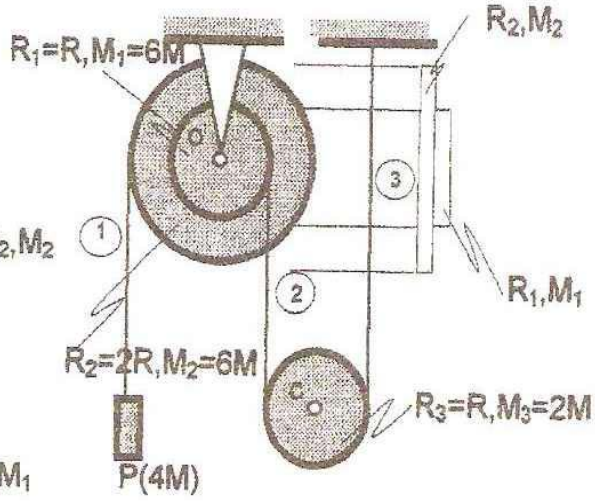


Fig.VIII.1.20.

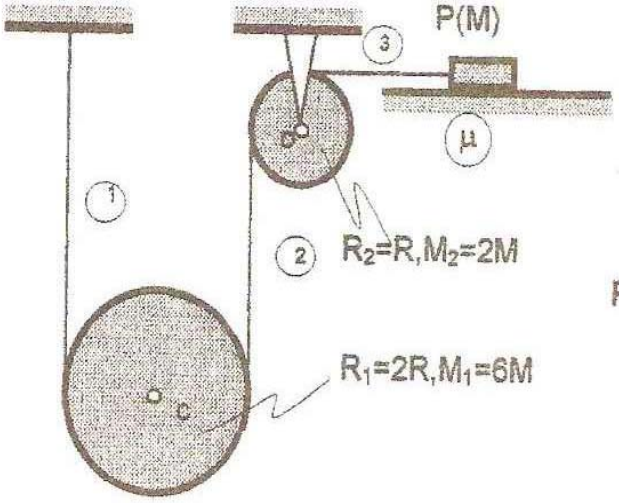
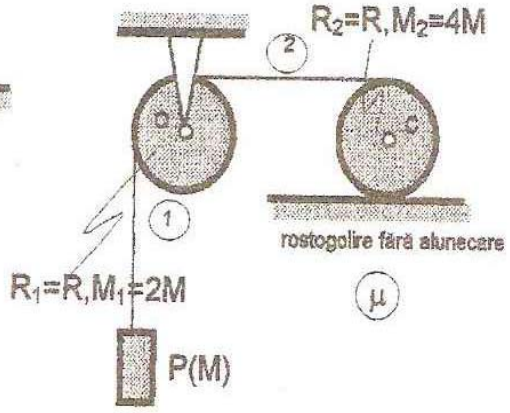


Fig.VIII.1.21.



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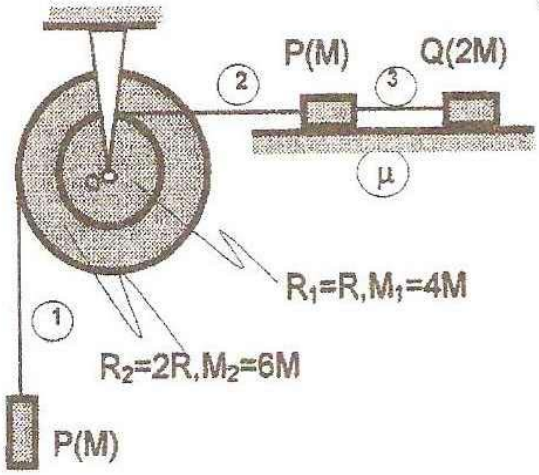


Fig.VIII.1.23.

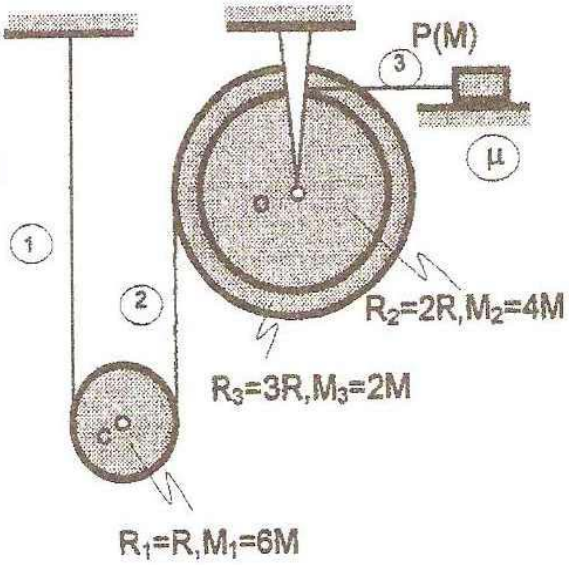


Fig.VIII.1.24.

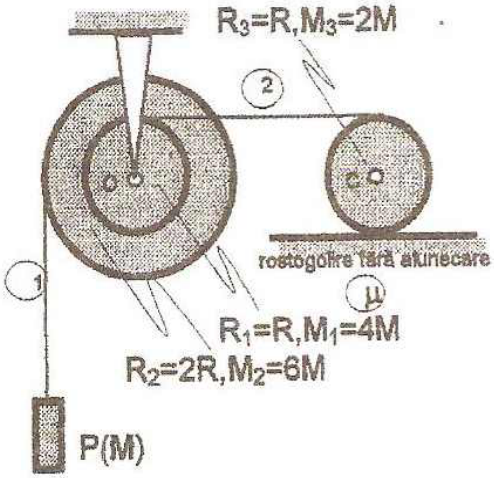


Fig.VIII.1.25.

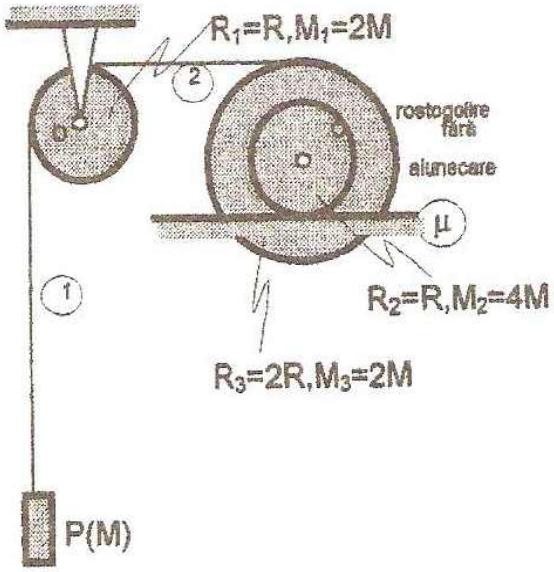


Fig.VIII.1.26.

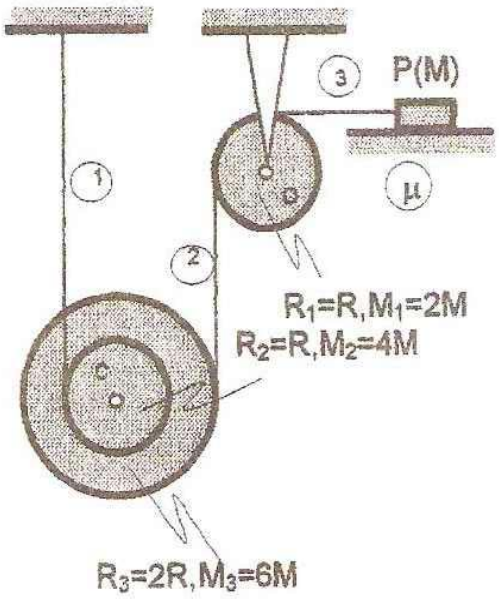
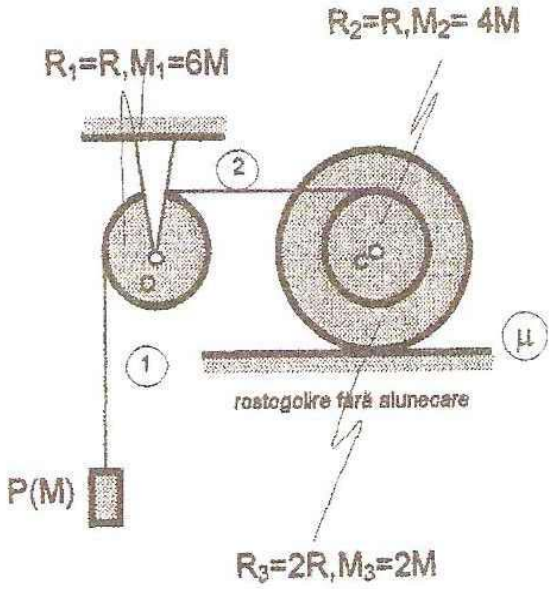


Fig.VIII.1.27.



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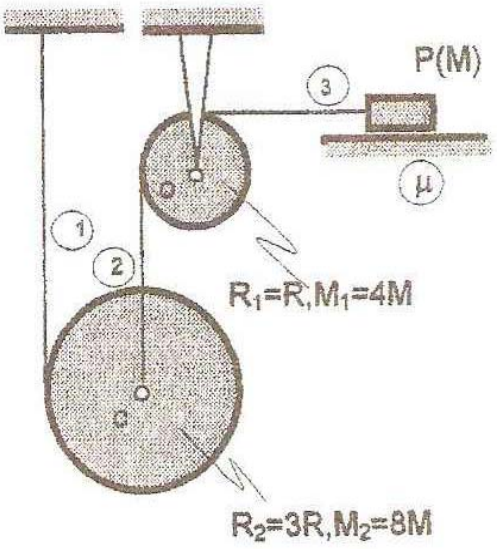

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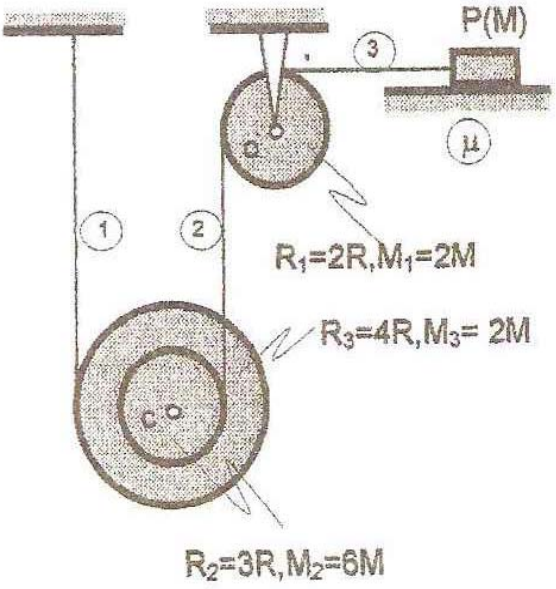


Fig.VIII.1.30.

VIII.2. STUDIUL MIŞCĂRII UNUI SISTEM FOLOSIND ECUAŢIILE LAGRANGE DE SPEŢA A II-A SAU TEOREMA ENERGIEI CINETICE

Problemele VIII.2.1. – VIII.2.30.

Folosind ecuaţiile Lagrange de speţa a II-a sau după caz teorema energiei cinetice să se scrie ecuaţiile diferenţiale ale mişcării sistemelor reprezentate în figurile VIII.2.1. – VIII.2.30.

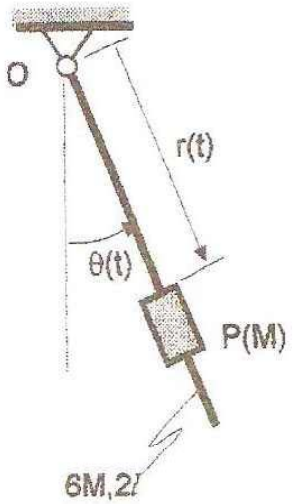


Fig.VIII.2.1.

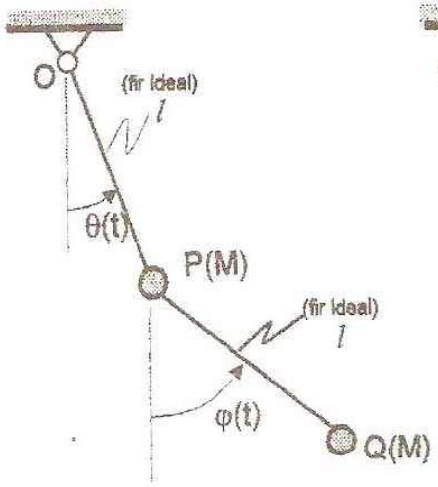
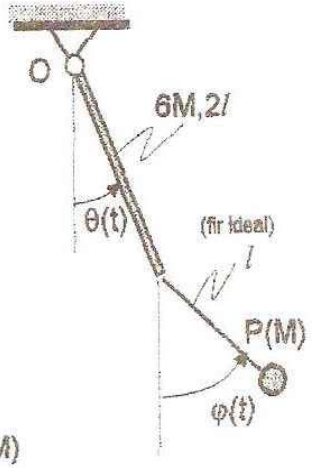


Fig.VIII.2.2.



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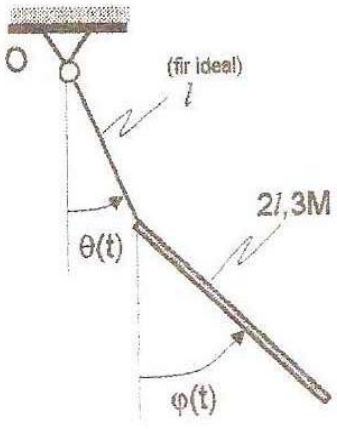

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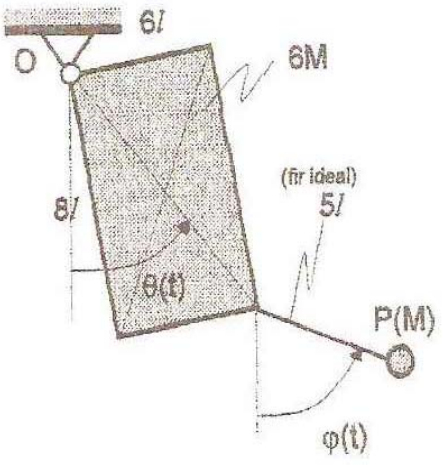

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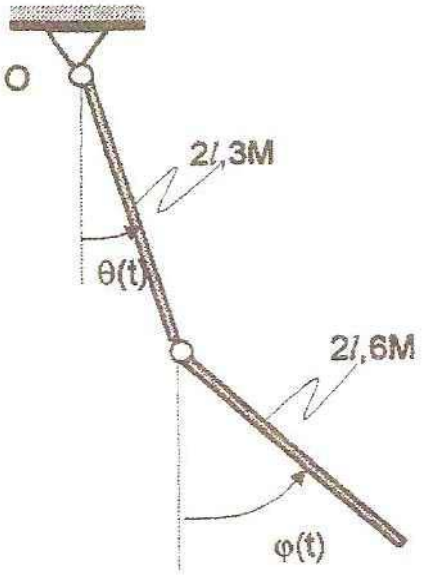


Fig.VIII.2.6.

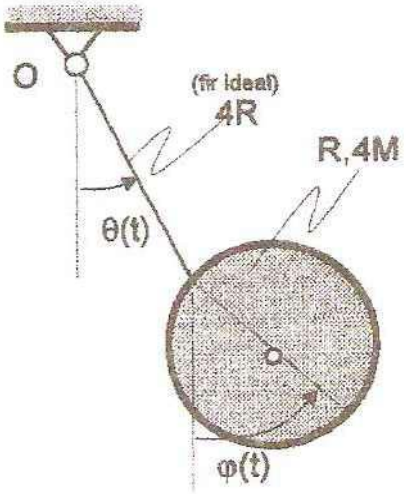


Fig.VIII.2.7.

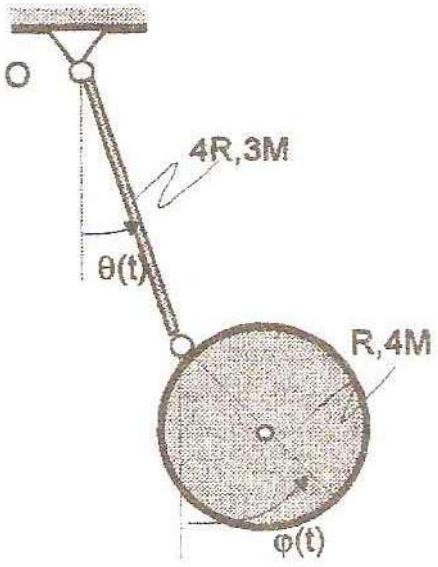
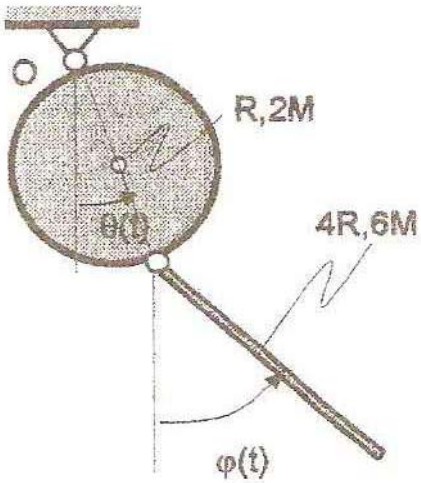


Fig.VIII.2.8.



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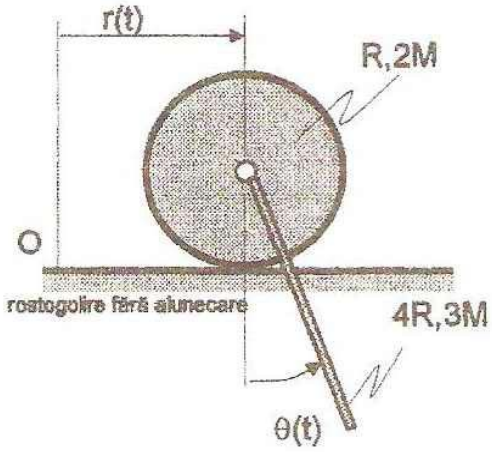


Fig.VIII.2.10.

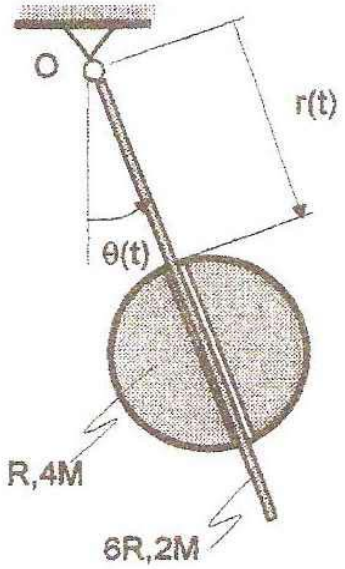


Fig.VIII.2.11.

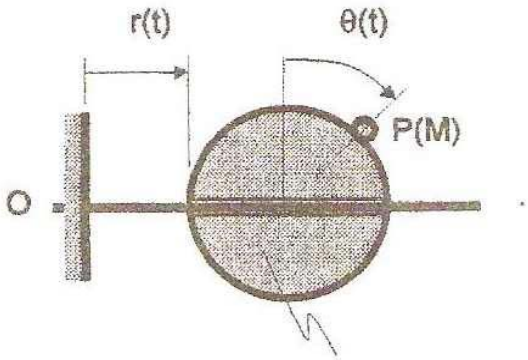


Fig.VIII.2.12.

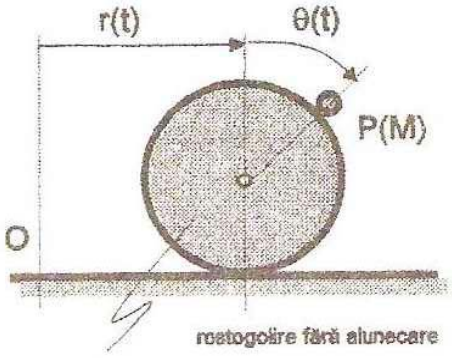
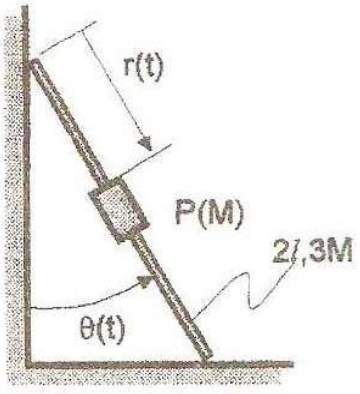


Fig.VIII.2.13.



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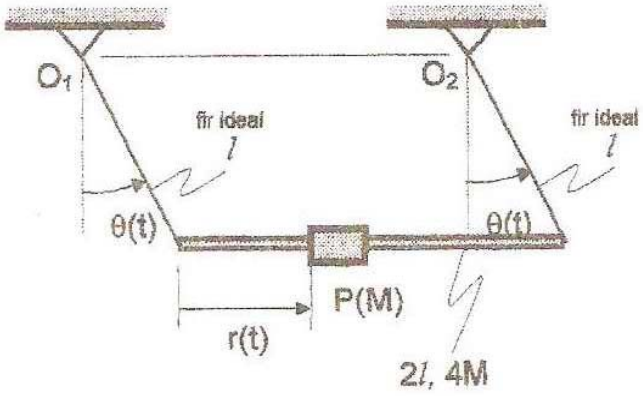

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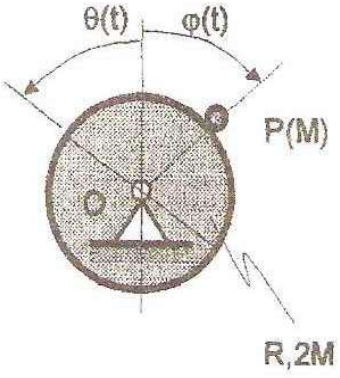

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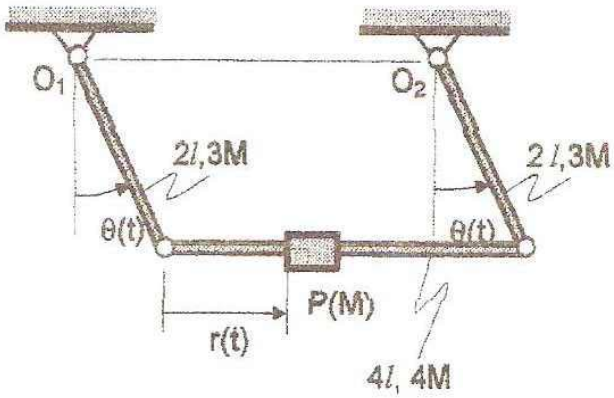

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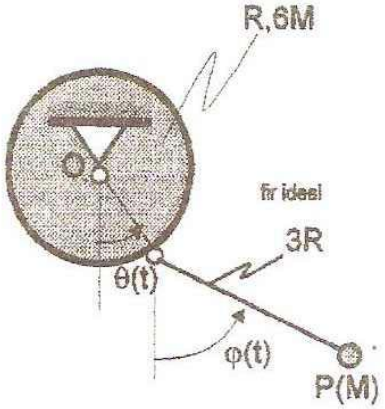

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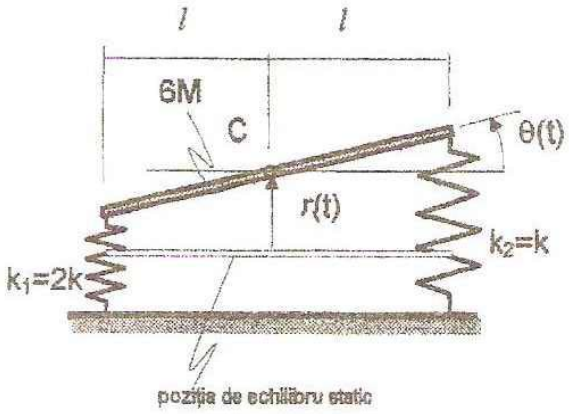

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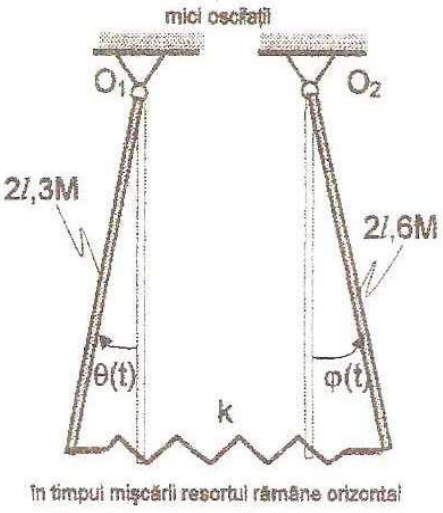

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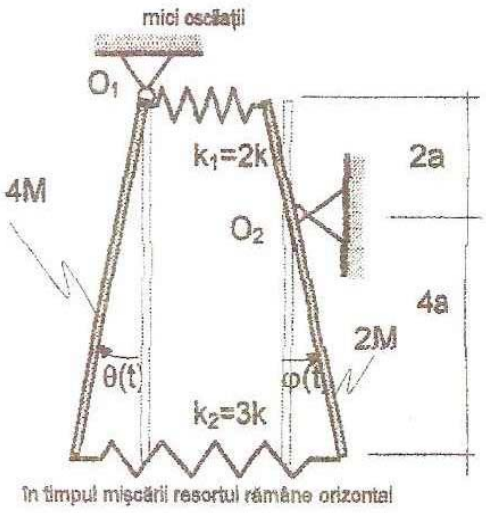
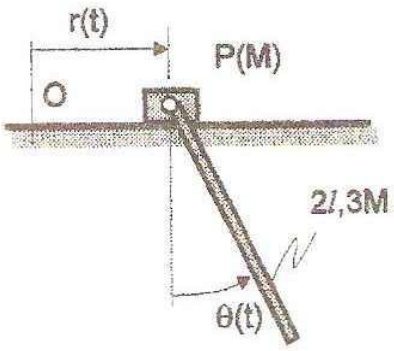


Fig.VIII.2.21.



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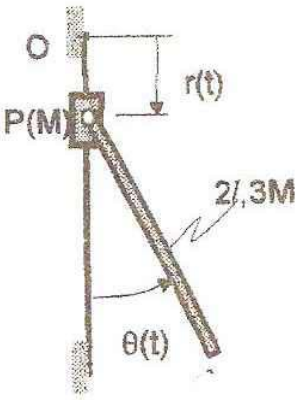

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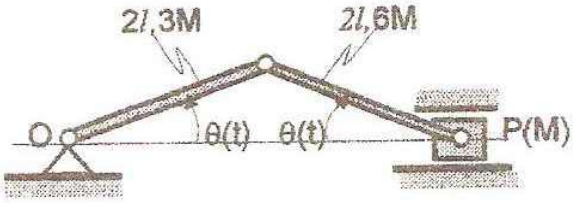

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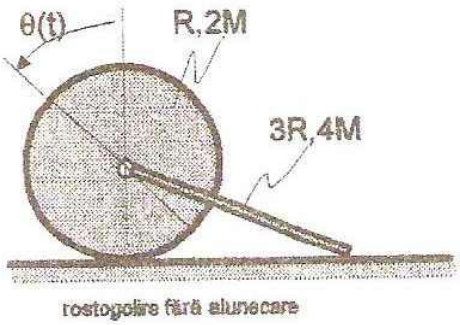

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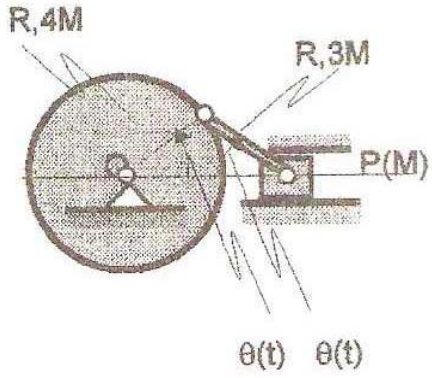

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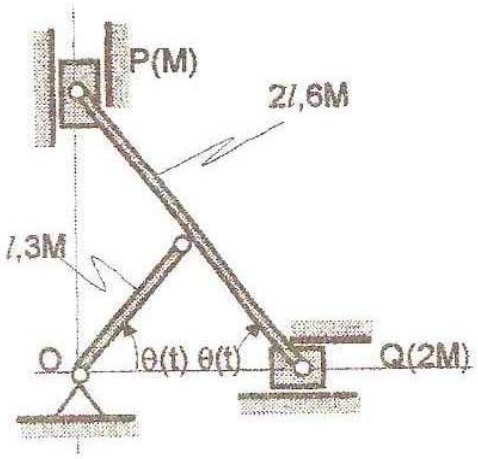
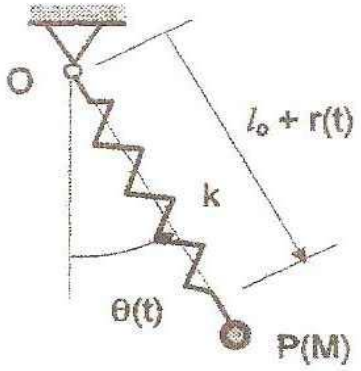


Fig.VIII.2.27



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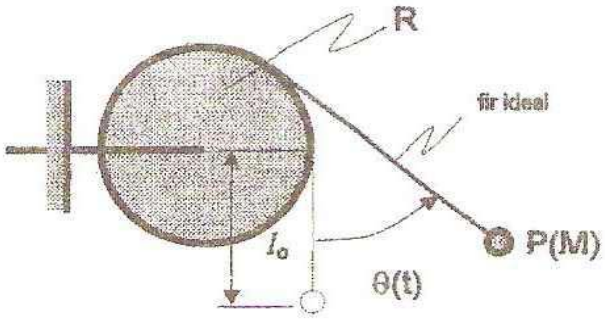


Fig.VIII.2.29.

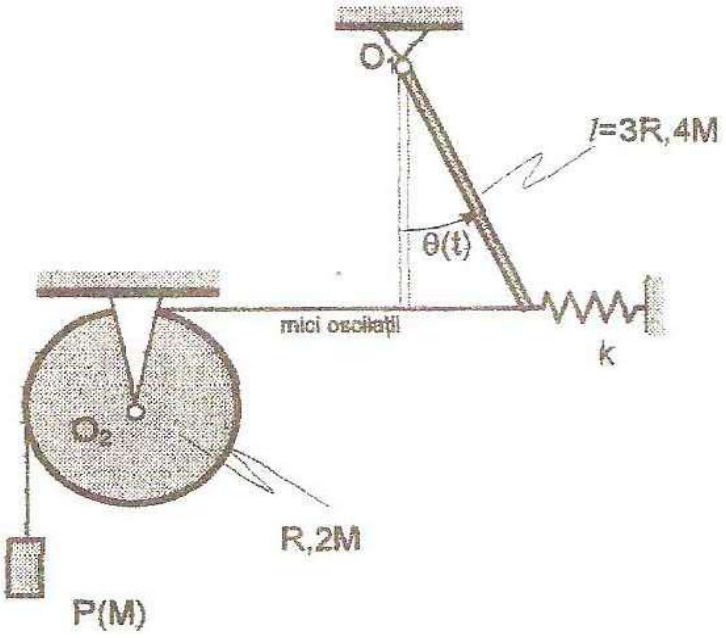


Fig.VIII.2.30.

SOLUȚII

CAPITOLUL I. REDUCEREA SISTEMELOR DE FORȚE OARECARE

I.1.

$\overline{R}(-2F;2F;-4F)$; $\overline{M}_O(-3aF;14aF;10aF)$; $M_{min}=1,22aF$;

A.C. $\begin{cases} 4x-4y-4z=11a \\ x+5y+2z=2a \end{cases}$ A(0 ;2,5a ;-5,25a) ; B(2,5a ;0 ;-0,25a) C(2,625a ;-0,125a ;0)

I.2.

$\overline{R}(5F;6F;-3F)$; $\overline{M}_O(-23aF;25aF;22aF)$; $M_{min}=-3,7aF$;

A.C. $\begin{cases} 15x+18y+61z=263a \\ 30x-34y-18z41a \end{cases}$ A(0 ;-4,13a ;-5,53a) ; B(3,46a ;0 ;3,46a) C(9,22a ;6,93a ;0)

I.3.

$\overline{R}(F;F;-2F)$; $\overline{M}_O(7aF;7aF;13aF)$; $M_{min}=-4,9aF$;

A.C. $\begin{cases} x+y+z=0 \\ x-5y-2z=27a \end{cases}$ A(0 ;-9a ;9a) ; B(9a ;0 ;-9a) C(4,5a ;-4,5a ;0)

I.4.

$\overline{R}(-5F;0;-4F)$; $\overline{M}_O(-12aF;40aF;15aF)$; $M_{min}=0$;

A.C. $\begin{cases} -4x+5z= -40a \\ y=3a \end{cases}$ A(0 ;3a ;-8a) ; B(10a ;3a ;0)

I.5.

$\overline{R}(-2F;-2F;-6F)$; $\overline{M}_O(-4aF;8aF;4aF)$; $M_{min}=-4,82aF$;

A.C. $\begin{cases} 3x+3y-2z=6a \\ x-10y+3z=-8a \end{cases}$ A(0 ;7,09a ;-7,63a) ; B(0,18a ;0 ;-2,73a) C(1,09a ;0.91a ;0)

I.6.

$\overline{R}(F;4F;-7F)$; $\overline{M}_O(-69aF;42aF;aF)$; $M_{min}=11,33aF$;

A.C. $\begin{cases} 7x+28y+17z=318a \\ 4x-50y-28z= -482a \end{cases}$ A(0 ;-10,81a ;36,53a) ; B(2,69a ;0 ;17,6a) C(5,19a ;10,06a ;0)

I.7.

$\overline{R}(-F;0;2F)$; $\overline{M}_O(0;26aF;16aF)$; $M_{min}=14,31aF$;

A.C. $\begin{cases} x+z= -26a \\ y=3,2a \end{cases}$ A(0 ;3,2a ;-26a) ; C(-26a ;3,2a ;0)

I.8.

$\overline{R}(-4F;F;-7F)$; $\overline{M}_O(-4aF;9aF;19aF)$; $M_{min}= -13,29aF$;

A.C. $\begin{cases} 28x-7y-17z=32a \\ 4x+65y+7z=104a \end{cases}$ A(0 ;1,88a ;-2,66a) ; B(7,55a ;0 ;10,55a) C(2,06a ;1,506a ;0)

I.9.

$\overline{R}(F;0;-10F)$; $\overline{M}_O(-9aF;4aF;-3aF)$; $M_{min}=2,09aF$;

A.C. $\begin{cases} 10x+z=4a \\ y=0,92a \end{cases}$ A(0 ;0,92a ;4a) ; C(0,4a ;0,92a ;0)

I.10.

$\overline{R}(3F;0;-8F)$; $\overline{M}_O(-4aF;12aF;-2aF)$; $M_{min}=0,47aF$;

A.C. $\begin{cases} 8x+3z=12a \\ y=0,47a \end{cases}$ A(0 ;0,47a ;4a) ; C(1,5a ;0,47a ;0)

I.11.

$\overline{R}(-4F;12F;-6F)$; $\overline{M}_O(-45aF;-18aF;18aF)$; $M_{min}= -10,29aF$;

A.C. $\begin{cases} 6x-18y-40z= -153a \\ 24x+26y+36z=171a \end{cases}$ A(0 ;3,4a ;2,295a) ; B(1,13a ;0 ;3,99a) C(-153a ;7,99a ;0)

I.12. $\overline{R}(-9F;-7F;-3F)$; $\overline{M}_O(18aF;6aF;12aF)$; $M_{min}= -20,36aF$;

A.C. $\begin{cases} 27x-21y-130z=-72a \\ 21x-30y+7z= -18a \end{cases}$ A(0 ;0,757a ;0,676a) ; B(-0,97a ;0 ;0,35a) C(-2,03a ;-0,62a ;0)

I.13. $\overline{R}(3F;-15F;4F)$; $\overline{M}_O(48aF;8aF;-21aF)$; $M_{min}= -3,8aF$;

A.C. $\begin{cases} 2x-10y-39z= -124a \\ 9x+5y+12z= 51a \end{cases}$ A(0 ;6,67a ;1,47a) ; B(1,33a ;0 ;3,248a) C(-1,1a ;12,18a ;0)

I.14. $\overline{R}(-7F;F;4F)$; $\overline{M}_O(30aF;-12aF;21aF)$; $M_{min}= -16,99aF$;

A.C. $\begin{cases} 14x+2y-25z= -27a \\ 7x+65y-4z= 267a \end{cases}$ A(0 ;4,18a ;1,41a) ; B(57a ;0 ;33a) C(-2,55a ;4,38a ;0)

I.15. $\overline{R}(5F;0;6F)$; $\overline{M}_O(24aF;18aF;-16aF)$; $M_{min}= 3,07aF$;

A.C. $\begin{cases} 6x-5z=-18a \\ y= -3,67a \end{cases}$ A(0 ;-3,67a ;3,6a) ; C(-3a ;-3,67a ;0)

I.16. $\overline{R}(-6F;-2F;-15F)$; $\overline{M}_O(-50aF;30aF;24aF)$; $M_{min}= -7,37aF$;

A.C. $\begin{cases} 9x+3y-4z=28a \\ 4x-87y+10z= -298a \end{cases}$ A(0 ;2,87a ;-4,85a) ; B(-8,6a ;0 ;-26,35a); C(1,94a ;3,51a ;0)

I.17. $\overline{R}(0;5F;-F)$; $\overline{M}_O(-24aF;-4aF;6aF)$; $M_{min}= -5,1aF$;

A.C. $\begin{cases} x = a \\ y+z= 24a \end{cases}$ B(a ;0 ;24a); C(a ;24a ;0)

I.18. $\overline{R}(12F;-12F;)$; $\overline{M}_O(32aF;-12aF;-68aF)$; $M_{min}= 31,1aF$;

A.C. $\begin{cases} 3x+3y=17a \\ 6z= 5a \end{cases}$ A(0 ;5,67a ;0,83a) ; B(5,67a ;0 ;0,83a)

I.19. $\overline{R}(3F;9F;-15F)$; $\overline{M}_O(-24aF;12aF;-3aF)$; $M_{min}= 4,71aF$;

A.C. $\begin{cases} 6x+18y+15z=41a \\ 3x-17y-12z= -33a \end{cases}$ A(0 ;0,07a ;2,64a) ; B(0,08a ;0 ;2,77a); C(0,77a ;2,08a ;0)

I.20. $\overline{R}(4F;-20F;-F)$; $\overline{M}_O(28aF;5aF;-60aF)$; $M_{min}= -7,37aF$;

A.C. $\begin{cases} 9x+3y-4z=28a \\ 4x-87y+10z= -298a \end{cases}$ A(0 ;2,87a ;-4,85a) ; B(-8,6a ;0 ;-26,35a); C(1,94a ;3,51a ;0)

I.21. $\overline{R}(-9F;-8F;F)$; $\overline{M}_O(32aF;-40aF;2aF)$; $M_{min}= 2,81aF$;

A.C. $\begin{cases} 72x-82y-z= -50a \\ 9x+8y+145z= 616a \end{cases}$ A(0 ;0,66a ;4,21a) ; B(-0,63a ;0 ;4,29a); C(38,1a ;34,1a ;0)

I.22. $\overline{R}(-5F;-F;3F)$; $\overline{M}_O(63aF;-50aF;10aF)$; $M_{min}= -39,72aF$;

A.C. $\begin{cases} 15x+28y+z=313a \\ 5x-34y-3z= -239a \end{cases}$ A(0 ;8,53a ;-78,28a) ; B(14a ;0 ;103a); C(6,08a ;7,92a ;0)

I.23.

$\overline{R}(2F;0;0)$; $\overline{M}_O(0;8aF;-4aF)$; $M_{min}= 0$;

A.C. $\begin{cases} y = 2a \\ z = 4a \end{cases}$ A(0 ;2a ;4a)

I.24.

$\overline{R}(-3F;20F;-7F)$; $\overline{M}_O(-44aF;45aF;36aF)$; $M_{min}= 36,45aF$;

A.C. $\begin{cases} 21x-140y-409z=745a \\ 30x+29y+70z= 208a \end{cases}$ A(0 ;-51,85a ;24,49a) ; B(10a ;0 ;-1,31a); C(10,54a ;-3,74a ;0)

I.25.

$\overline{R}(-6F;3F;-6F)$; $\overline{M}_O(-18aF;-12aF;12aF)$; $M_{min}= 0$;

A.C. $\begin{cases} 4x-2y-5z= -14a \\ x+4y+z= 10a \end{cases}$ A(0 ;2a ;2a) ; B(4a ;0 ;6a); C(-2a ;3a ;0)

I.26.

$\overline{R}(5F;-10F;5F)$; $\overline{M}_O(22aF;6aF;-10aF)$; $M_{min}= 0$;

A.C. $\begin{cases} x-2y-5z= -10a \\ 5x+5y+5z= 16a \end{cases}$ A(0 ;2a ;1,2a) ; B(a ;0 ;2,2a); C(-1,2a ;4,4a ;0)

I.27.

$\overline{R}(-4F;20F;-23F)$; $\overline{M}_O(-92aF;-4aF;16aF)$; $M_{min}= -2,6aF$;

A.C. $\begin{cases} 23x-115y-104z= -464a \\ 16x+109y+92z= 436a \end{cases}$ A(0 ;3,58a ;0,5a) ; B(10,44a ;0 ;2,92a); C(9,93a ;2,54a ;0)

I.28.

$\overline{R}(-15F;-10F;-F)$; $\overline{M}_O(20aF;-6aF;0)$; $M_{min}= -2,22aF$;

A.C. $\begin{cases} 3x+2y-65z= -58a \\ 75x-113y+5z= 10a \end{cases}$ A(0 ;-0,05a ;0,89a) ; B(0,07a ;0 ;0,9a); C(-13,36a ;-8,96a ;0)

I.29.

$\overline{R}(-F;-7F;11F)$; $\overline{M}_O(28aF;-10aF;-6aF)$; $M_{min}= -1,84aF$;

A.C. $\begin{cases} 11x+77y+50z= 206a \\ 7x-122y-77z= -302a \end{cases}$ A(0 ;-4,53a ;11,11a) ; B(0,64a ;0 ;3,98a); C(a ;2,53a ;0)

I.30.

$\overline{R}(-4F;-3F;8F)$; $\overline{M}_O(30aF;-24aF;-6aF)$; $M_{min}= -10,18aF$;

A.C. $\begin{cases} 32x+24y+25z= 186a \\ 3x-20y-6z= -54a \end{cases}$ A(0 ;0,66a ;6,81a) ; B(-0,87a ;0 ;8,56a); C(3,4a ;3,21a ;0)

I.31.

$\overline{R}(4F;-9F;6F)$; $\overline{M}_O(30aF;12aF;-24aF)$; $M_{min}= -11,45aF$;

A.C. $\begin{cases} 24x-54y-97z= -318a \\ 18x+26y+27z= 138a \end{cases}$ A(0 ;4,39a ;0,88a) ; B(2,01a ;0 ;3,78a); C(-0,49a ;5,67a ;0)

I.32.

$\overline{R}(4F;8F;-15F)$; $\overline{M}_O(-72aF;72aF;32aF)$; $M_{min}= -11aF$;

A.C. $\begin{cases} 60x-225y-104z= -792a \\ 15x+30y+20z= 216a \end{cases}$ A(0 ;-4,94a ;18,4a) ; B(2,4a ;0 ;9a); C(4,8a ;4,8a ;0)

I.33.

$\overline{R}(4F;0;-12F)$; $\overline{M}_O(-18aF;12aF;0)$; $M_{min}= -5,69aF$;

A.C. $\begin{cases} y = 1,35a \\ 3x+z= 3a \end{cases}$ A(0 ;1,35a ;3a) ; C(a ;1,35a ;0)

I.34.

$\overline{R}(9F;-4F;F)$; $\overline{M}_O(16aF;20aF;-24aF)$; $M_{min}= 4,04aF$;

A.C. $\begin{cases} 9x-4y-97z= -244a \\ 18x+41y+2z= 116a \end{cases}$ A(0 ;2,71a ;2,4a) ; B(6,1a ;0 ;3,08a); C(-21,63a ;12,33a ;0)

I.35.

$\overline{R}(F;9F;-20F)$; $\overline{M}_O(-45aF;20aF;0)$; $M_{min}= -11aF$;

A.C. $\begin{cases} 20x+180y+82z= 425a \\ 9x-401y-180z= -900a \end{cases}$ A(0 ;-5,44a ;17,14a) ; B(0,62a ;0 ;5,03a); C(0,88a ;2,26a ;0)

I.36.

$\overline{R}(-F;-10F;-F)$; $\overline{M}_O(2aF;-2aF;-12aF)$; $M_{min}= 2,97aF$;

A.C. $\begin{cases} x+10y-101z= -22a \\ x+z= -a \end{cases}$ A(0 ;7,9a ;-a) ; B(-1,21a ;0 ;0,21a); C(-a ;-2,1a ;0)

I.37.

$\overline{R}(-F;-7F;2F)$; $\overline{M}_O(20aF;-8aF;-6aF)$; $M_{min}= 3,27aF$;

A.C. $\begin{cases} x+7y+25z= 74a \\ 7x-5y-14z= -34a \end{cases}$ A(0 ;-6,91a ;4,89a) ; B(0,98a ;0 ;2,92a); C(2,44a ;10,22a ;0)

I.38.

$\overline{R}(5F;-7F;11F)$; $\overline{M}_O(42aF;-25aF;-35aF)$; $M_{min}= 0$;

A.C. $\begin{cases} 55x-77y-74z= -169a \\ 35x+146y+77z= 637a \end{cases}$ A(0 ;6,92a ;-4,93a) ; B(5a ;0 ;6a); C(2,27a ;3,82a ;0)

I.39.

$\overline{R}(0;-3F;0)$; $\overline{M}_O(12aF;0;-4aF)$; $M_{min}= 0$;

A.C. $\begin{cases} X= -1,33a \\ z= 4a \end{cases}$ B(-1,33a ;0 ;4a);

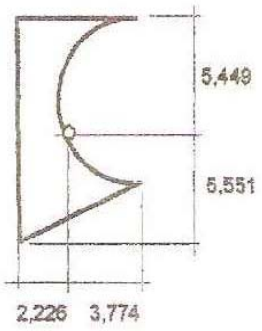
I.40.

$\overline{R}(15F;-4F;8F)$; $\overline{M}_O(0;36aF;-36aF)$; $M_{min}= -24,74aF$;

A.C. $\begin{cases} 120x-32y-241z= -540a \\ 60x+289y+32z= 540a \end{cases}$ A(0 ;1,64a ;2,02a) ; B(6,17a ;0 ;5,31a); C(-3,79a ;2,66a ;0)

CAPITOLUL II. CENTRE DE GREUTATE PENTRU PLĂCI PLANE OMOGENE

II.1



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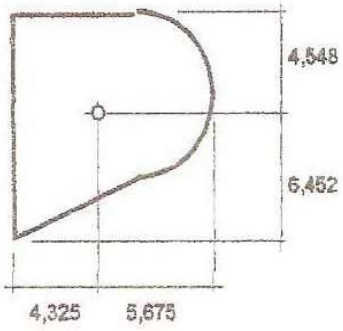

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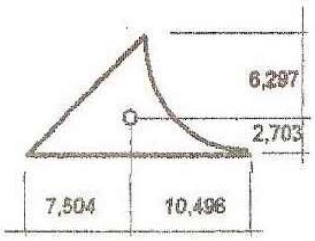

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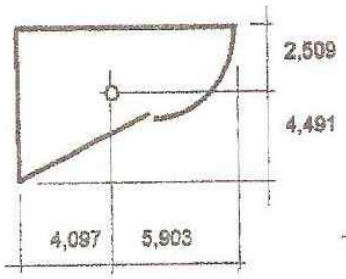

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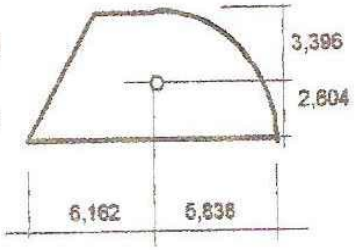

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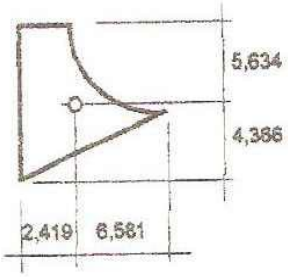

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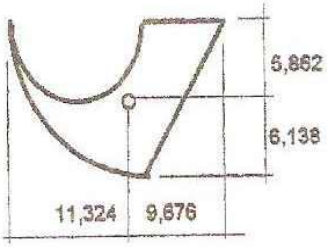

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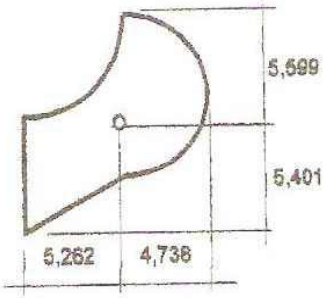

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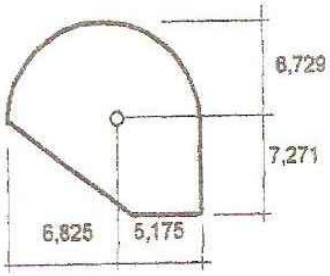

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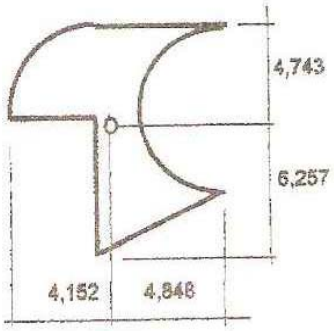

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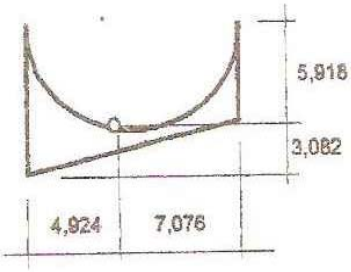

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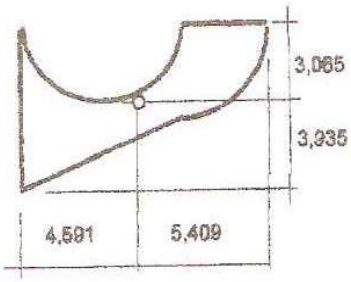

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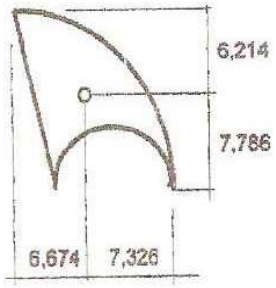

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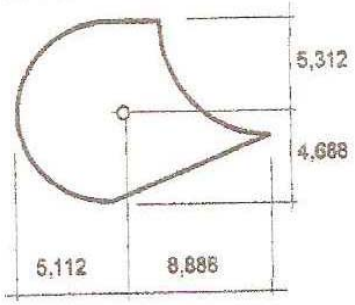

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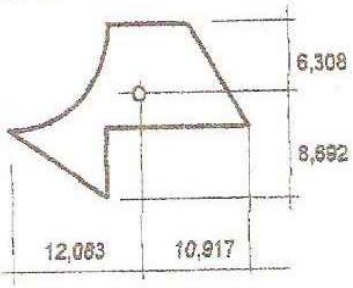

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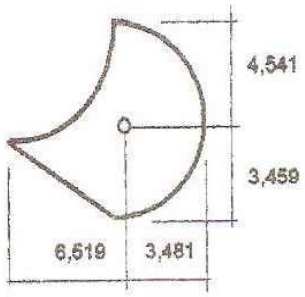

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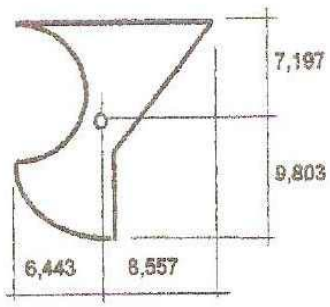

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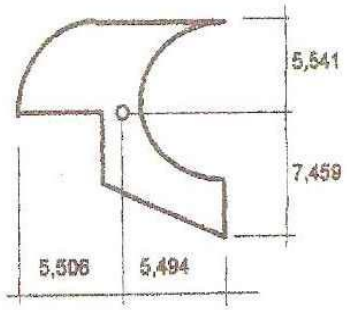

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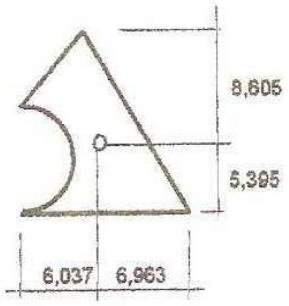

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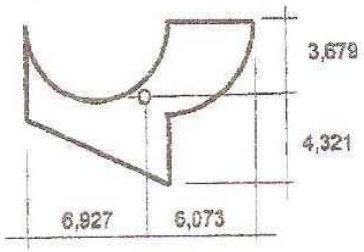

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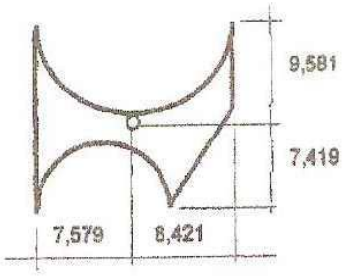

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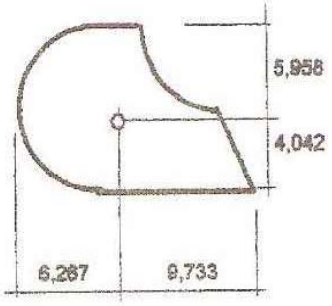

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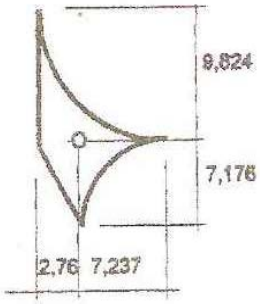

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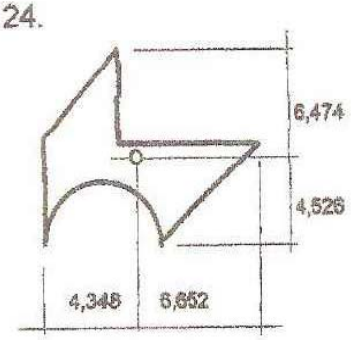

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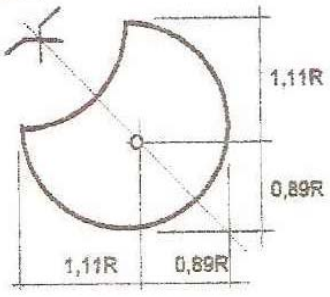

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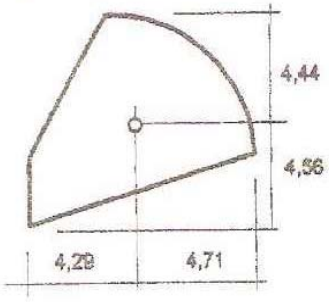

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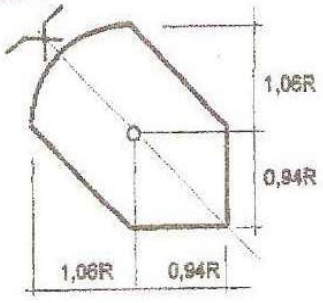

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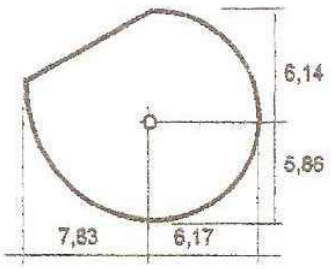

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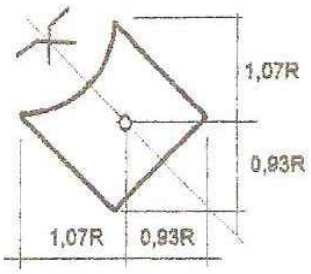

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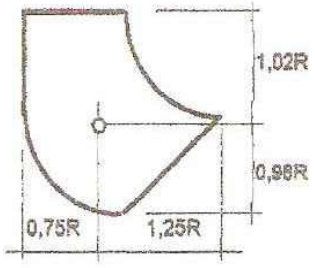

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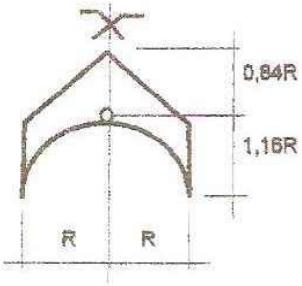

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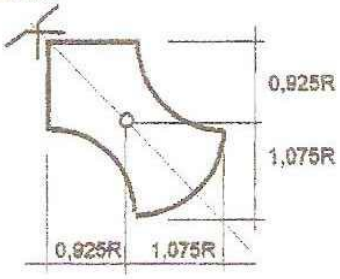

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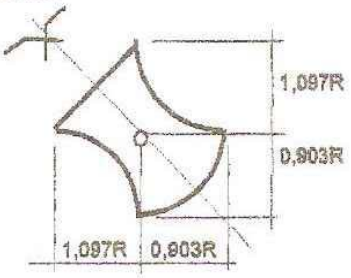

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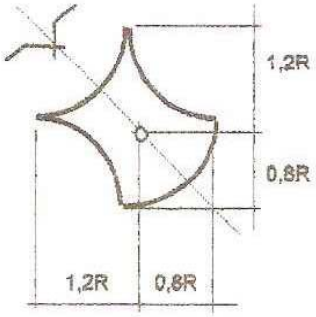

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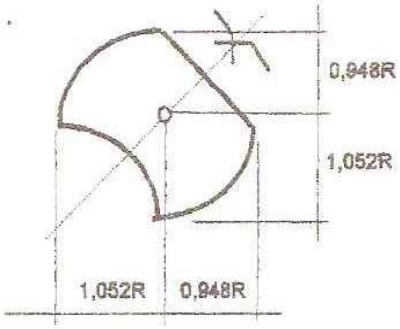
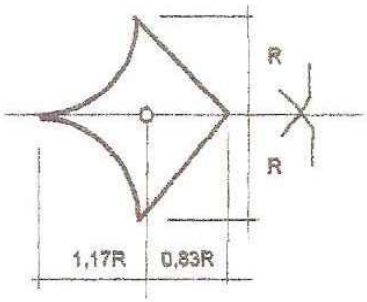


II.36.



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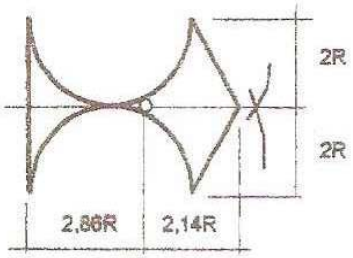

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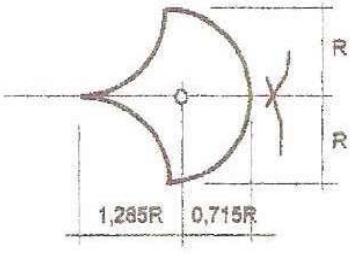

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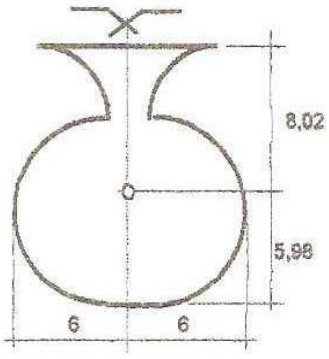

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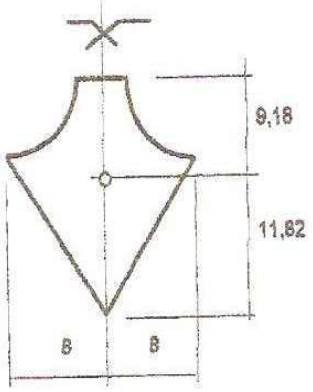


CAPITOLUL III. CALCULUL REACȚIUNILOR LA CADRE

III.1. V_A= 63,33 kN↑ ,H_A = 13,33 kN→ , M_A = 139,94 kNm↻ ,V_D = 71,67 kN↑ , H_D = 16,67 kN → , V_F = 35 kN↑;

III.2. V_A= 79,29 kN ↑,H_A = 32,15 kN← , M_A = 330,06 kNm ↻ ,V_D = 200,71 kN ↑ , H_D = 1,18 kN ← , H_F = 63,33 kN→ ;

III.3. V_A= 145,55kN ↑,H_A = 102,22 kN→ ,V_D = 134,45 kNm↑ ,H_D = 87,22 kN ← , M_D = 889,9 kN ↻ , V_F = 40 kN↑

III.4. V_A= 138,33 kN ↑,H_A = 85 kN→ , V_D = 181,67 kNm ↑ ,H_D = 88,33 kN← , M_D = 1413,28 kN ↻ , H_F = 63,33 kN→

III.5. V_A= 30,45 kN ↑ ,H_A = 10,91 kN ← , V_D = 32,74 kNm ↑ ,V_F = 66,81 kN ↑ , H_F = 19,09 kN ← , M_F = 259,94 kN↺

III.6. V_A= 45,83 kN ↑ ,H_A = 15 kN → , M_A = 16,7 kNm ↺ ,V_C = 130,42 kN ↑ , V_F = 43,75 kN ↑ , H_F = 30 kN →

III.7. V_A= 36,78 kN ↑ ,H_A = 5,36 kN ← , V_D = 73,22 kNm ↑ ,H_D = 16,31 kN ←, M_D = 271,36 kN ↻ , H_F = 23,33 kN ←

III.8. V_A= 90 kN↑ ,V_C = 50 kN↑, H_C = 20 kN→ ,V_F = 140 kN ↑ , H_F = 80 kN ← , M_F = 320 kNm↺

III.9. V_A= 197,5 kN↑ ,H_A = 40 kNm ← ,V_D = 43,75 kN ↑ ,V_F = 178,75 kN ↑ , H_F = 20 kN ← , M_F = 830 Kn↺

III.10. V_A= 60 kN↑ ,V_D = 37,22 Kn↑, H_D = 21,67 kN→ ,V_F = 2,78 kN↑, H_F = 66,67 kN← , M_F = 537,76 kNm ↻

III.11. V_A= 30 kN ↑, H_A = 60 kN →, M_A = 510 kNm ↺, V_C = 225 kN ↑,
V_E = 105 kN ↑.

III.12. V_A= 52,5 kN ↑, H_C = 20,62 kN ←, V_E = 67,5 kN ↑, H_E = 9,38 kN ←,
M_E = 305 kNm ↺.

III.13. V_A= 20 kN ↑, V_C = 146,25 kN ↑, H_C = 45 kN →, M_C = 382,5 kNm ↺,
V_E = 73,75 kN ↑.

III.14. V_A= 68 kN ↑, V_C = 7,5 kN ↑, V_E = 64,5 kN ↑, H_E = 45 kN ←,
M_E = 10 kN m ↻.

III.15. H_A= 4,29 kN →, V_B = 111,43 kN ↑, V_D = 88,57 kN ↑, H_D = 15,71 kN →,
V_F = 20 kN ↑, H_F = 40 kN →

III.16. V_A= 30 kN ↑, V_D = 85 kN ↑, H_D = 15 kN →, M_D = 450 kNm ↺,
V_F = 245 kN ↑, H_F = 30 kN →

III.17. V_A= 3,33 kN ↑, V_C = 79,17 kN ↑, H_C = 15 kN →, M_C = 161,7 kN m ↺,
V_F = 17,5 kN ↑, H_F = 30 kN →

III.18. V_A= 37,12 kN ↑, H_A = 0,91 kN ←, V_D = 30,21 kN ↑, V_F = 52,67 kN ↑,
H_F = 45,91 kN →, M_F = 235,5 kNm ↺

III.19. V_A= 70kN ↑, V_C = 55,84kN ↑, H_C = 11,67kN →, V_F = 54,16kN ↑,
H_F = 56,67kN ←, M_F = 340kNm ↻

III.20. V_A= 14kN ↑, H_A = 10,75kN ←, V_D = 39,33kN ↑, H_D =4,25 kN ←,
V_F = 66,67kN ↑, H_F = 30kN ←

III.21. V_A= 84,83kN ↑, H_A = 37,78kN →, M_A = 19,1kNm ↺, V_D = 108,5kN ↑,
H_D = 22,22kN →, V_F = 46,67kN ↑

III.22. V_A= 42,5daN ↑, V_C = 42,5daN ↑, V_E = 30daN ↓, V_F = 65daN ↑, H_F = 45daN ←

III.23. V_A= 10daN ↑, V_C = 20daN ↑, V_E = 10N ↓, V_G = 40daN ↑, H_G = 30N ←,
M_G = 120daNm ↻

III.24. V_A= 25kN ↓, V_C = 135kN ↑, H_C = 30kN ←, M_C = 70kNm ↻, H_E = 45kN ←

III.25. V_A= 62,5kN ↑, H_A = 20kN ←, V_D = 33,75kN ↑, H_D = 10kN ←,
M_D = 205kNm ↻, V_F =13,75 kN ↑

III.26. V_A= 125kN ↑, H_A = 15kN ←, H_D = 7,5kN ←, V_F = 75kN ↑, H_F = 22,5kN ←,
M_A = 5kNm ↻

III.27. V_A= 5daN ↑, V_C = 45daN ↑, H_C = 10daN ←, V_E = 20daN ↑, H_E = 40daN →.

III.28. V_A= 210kN ↑, H_A = 223,33kN →, V_D = 40kN ↑, H_D = 148,33kN ←, M_A = 1130kNm ↻.

III.29. V_A= 46,67kN ↑, V_C = 393,41kN ↑, H_C = 60kN →, V_E = 386,75kN ↓, V_F = 166,67kN ↑.

III.30. V_A= 55kN ↑, V_C =5kN ↓, V_E = 2,5kN ↑, H_E = 45kN ←, V_F = 47,5kN ↑.

III.31. V_A= 15kN ↑, V_C = 40,83kN ↑, H_C = 14,375kN ←, V_E = 54,17kN ↑, H_E = 15,625kN ←.

III.32. V_A= 45kN ↑, V_C = 90kN ↑, V_E = 310kN ↑, V_F = 115kN ↓, H_F = 60kN →.

III.33. Corpul CBD prezintă o formă critică: direcția reazemului simplu din C trece prin articulația intermediară B (care pentru corpul CBD devine articulație fixă). În consecință reacțiunile sistemului nu pot fi determinate folosind ecuațiile de echilibru static.

III.34. V_A= 45kN ↑, H_A = 58,33kN →, V_B = 10kN ↓, V_E = 135kN ↑, H_E = 103,33kN ←.

III.35. V_A= 30daN ↑, H_A = 120daN →, M_A = 640daNm ↺, V_D = 120daN ↑, H_D = 60daN ←.

III.36. V_A= 75kN ↑, H_A = 30kN →, M_A = 10kNm ↻, V_D = 86,25kN ↑, V_E = 58,75kN ↑

III.37. V_A= 203,75daN ↑, H_A = 15daN →, M_A = 25daNm ↺, V_D =166,25daN ↑, H_D = 30daN →.

III.38. V_A= 93,33kN ↓, H_A = 10kN →, V_B = 240kN ↑, V_E = 33,33kN ↑, H_E = 20kN →.

III.39. V_A= 15kN ↑, H_A = 30kN →, V_B = 150kN ↑, V_D = 180kN ↑, V_F = 45kN ↑.

III.40. V_A= 78,75daN ↑, H_A = 45daN →, V_B = 26,25daN ↓, V_D = 25daN ↑, V_F = 112,5daN. ↑

CAPITOLUL IV. CALCULUL EFORTURILOR ÎN BARELE GRINZILOR CU ZĂBRELE SIMPLE

IV.1. N_{1-2}= -100 kN, N_{1-3} = -45 kN, N_{2-3} = 125 kN, N_{2-4} = -30 kN, N_{3-4} = -40 kN,

N_{3-5} = 30 kN, N_{6-8} = -90 kN, N_{7-8} = 50 kN, N_{7-9} = 60 kN.

IV.2. N_{1-2}= -60 kN, N_{1-3} = 45kN, N_{2-3} = 75 kN, N_{2-4} = -90 kN, N_{3-4} = -60 kN, N_{3-5} = 90 kN, N_{6-8} =-135 kN, N_{7-8} = 75 kN, N_{7-9} = 90 kN.

IV.3. N_{1-2}= -125 kN, N_{1-3} = 30 kN, N_{2-3} =60 kN, N_{2-4} = -60 kN, N_{3-5} =30 kN, N_{2-5} =50 kN, N_{6-8} = -60 kN, N_{6-9} = -50 kN, N_{7-9} = 90 kN.

IV.4. N_{1-2}= -75 kN, N_{1-3} = 90 kN, N_{2-3} = 60 kN, N_{2-4} = -90 kN, N_{2-5} = 0, N_{3-5} = 90 kN, N_{6-8} = -60 kN, N_{6-9} = -50 kN, N_{7-9} =90 kN.

IV.5. N_{1-2}= -36,1 kN, N_{1-3} = -24 kN, N_{2-3} = 20 kN, N_{2-4} = -36,1 kN, N_{3-4} = -24 kN, N_{3-5} = 20 kN, N_{6-8} = -43,25 kN, N_{7-8} = -47,15 kN, N_{7-9} = 72 kN.

IV.6. N_{1-2}= -7,5 kN, N_{1-3} = 56 kN, N_{2-3} = 4,5 kN, N_{2-4} = -56 kN, N_{3-4} = 59,16 kN, N_{3-5} = 8,66 kN, N_{6-8} = 132,99 kN, N_{6-9} = -112,91 kN, N_{7-9} = -38,67 kN.

IV.7. N_{1-2}= -50 kN, N_{1-3} = -60 kN, N_{2-3} = 0, N_{2-4} = -40 kN, N_{3-4} = -12,5 kN, N_{4-5} = 7,5 kN, N_{6-8} = -31,65kN, N_{7-8} = -84,86kN, N_{7-9} =90 kN.

IV.8. N_{1-2}= 136,67 kN, N_{1-3} = -138,22kN, N_{2-3} = -85 kN, N_{2-4} = 246,38 kN, N_{3-4} = -3,33 kN, N_{3-5} = -240,38 kN, N_{6-8} = -145,41 kN, N_{7-8} = -84,86 kN, N_{7-9} = 190kN.

IV.9. N_{1-2}= 97,99 kN, N_{1-3} = -53,69 kN, N_{2-3} = 80 kN, N_{2-4} = 97,99 kN, N_{3-4} = -89,48 kN, N_{3-5} = 35,79 kN, N_{6-8} = 65,22 kN, N_{6-9} = 30,03 kN, N_{7-9} = -22 kN.

IV.10. N_{1-2}= -45,45 kN, N_{1-3} = -57,27 kN, N_{2-3} = 0,297 kN, N_{2-4} = 44,91 kN, N_{3-4} = -57,27 kN, N_{3-5} = 0,297 kN, N_{6-8} = -29,83 kN, N_{7-8} = -47,15 kN, N_{7-9} = 60 kN.

IV.11. N_{1-2}= -71,25 kN, N_{1-3} = 72,75 kN, N_{2-3} = 100 kN, N_{2-4} = -40,5 kN, N_{2-5} = -53,75kN, N_{3-5} = 72,75 kN, N_{6-8} = 2,4 kN, N_{6-9} = -53,75 kN, N_{7-9} = 8,25 kN.

IV.12. N_{1-2}= -35 kN, N_{1-3} = -9 kN, N_{2-3} = 50 kN, N_{2-4} = 25,5 kN, N_{2-5} = -27,5 kN, N_{3-5} = -9 kN, N_{6-8} = 58,5 kN, N_{6-8} = -27,5 kN, N_{7-9} = -42 kN.

IV.13. N_{1-2}= -23,75 kN, N_{1-3} = 74,25 kN, N_{2-3} = 50 kN, N_{2-4} = -51 kN, N_{2-5} = -38,75 kN, N_{3-5} = 74,25 kN, N_{6-8} = -27,75 kN, N_{7-8} = 38,75 kN, N_{7-9} = 4,5 kN.

IV.14. N_{1-2}= -126,67 kN, N_{1-3} = 61,33 kN, N_{2-3} = 100 kN, N_{2-4} = -29,33 kN, N_{2-5} = -40kN, N_{3-5} = 61,33 kN, N_{6-8} = 34,67 kN, N_{6-9} = -40 kN, N_{7-9} = -2,67 kN.

IV.15. N_{1-2}= 100daN, N_{1-3} = -60 daN, N_{2-3} = -130daN, N_{2-4} = -7,5daN,

N_{2-5} = 62,5daN, N_{3-5} = -30 daN, N_{6-8} = -45daN, N_{7-8} = 62,5daN, N_{7-9} = 7,5daN.

IV.16. N_{1-2}= 46,61daN, N_{1-3} = -84,13daN, N_{2-3} = -34daN, N_{2-4} = 23,79daN, N_{2-5} = 40,86daN, N_{3-5} = -76,61daN, N_{6-6} = 1,33daN, N_{7-8} = -40,86daN, N_{7-9} = -8,67daN.

IV.17. N_{1-2}= 40kN, N_{1-3} = -330 kN, N_{1-4} = -50kN, N_{2-4} = 379,74kN, N_{3-4} = 50kN, N_{4-6} = 348,09kN, N_{6-8} = 295,35kN, N_{6-8} = -12,03kN, N_{5-7} = -270kN.

IV.18. N_{1-2}= -160daN, N_{1-3} = -353,33daN, N_{2-3} = 192,31daN, N_{2-4} =246,79daN, N_{3-4} = -160daN, N_{3-5} = -246,79daN, N_{6-8} = 40daN, N_{6-9} = 36,05daN, N_{7-9} = -60daN.

IV.19. N_{1-2}= -113,15daN, N_{1-3} = 40daN, N_{2-3} = 60daN, N_{2-4} = -70,71daN, N_{2-5} = -42,43daN, N_{3-5} = 40daN, N_{6-8} = 0, N_{7-8} = -22,37daN, N_{7-9} = 10daN.

IV.20. N_{1-2}= -137,38daN, N_{1-3} = 64,3daN, N_{2-3} = 30daN, N_{2-4} = -91,74daN, N_{2-5} = -45,63daN, N_{3-5} = 64,3daN, N_{6-8} = -121,34daN, N_{6-9} = 37,28daN, N_{7-9} = 34,16daN.

IV.21. N_{1-2}= -126,35kN, N_{1-3} = 135,12kN, N_{2-3} = -60kN, N_{2-4} = -126,35kN, N_{3-4} = 75kN, N_{3-5} = 30,12kN, N_{6-8} = -144,4kN, N_{6-9} = 100,67kN, N_{7-9} = 75kN.

IV.22. N_{1-2}= -90daN, N_{1-3} = -135daN, N_{2-3} = 33,53daN, N_{2-4} = 134,28daN, N_{3-4} = -9,97daN, N_{3-5} = -120daN, N_{6-8} = 89,48daN, N_{6-7} = 42,43daN, N_{5-7} = -110daN.

IV.23. N_{1-2}= -100daN, N_{1-3} = -57,5daN, N_{2-3} = 106,18daN, N_{2-4} = 11,23daN, N_{3-4} = -50daN, N_{4-6} = 11,23daN, N_{6-8} = 67,11daN, N_{6-7} = 28,29daN, N_{5-7} = -80daN.

IV.24. N_{1-2}= -80daN, N_{1-3} = -145daN, N_{2-3} = 89,48daN, N_{2-4} = 105daN, N_{3-4} = -30daN, N_{4-5} = 105daN, N_{6-8} = 55daN, N_{7-8} = -55,92daN, N_{7-9} = -30daN.

IV.25 N_{1-2}= -86,46daN, N_{1-3} = 11,87daN, N_{2-3} = 86,46daN, N_{2-4} = -63,75daN, N_{3-4} = 13,54daN, N_{3-5} = 55,62daN, N_{6-8} = -31,24daN, N_{6-7} = -13,54daN, N_{5-7} = 39,37daN.

IV.26. N_{1-2}= -45daN, N_{1-3} = 40daN, N_{2-3} = 54,08daN, N_{2-4} = -69,96daN, N_{3-4} = 15daN, N_{3-5} = 69,96daN, N_{6-8} = -55,92daN, N_{7-8} = -14,14daN, N_{7-9} = 60daN.

IV.27. N_{1-2}= -120,81kN, N_{1-3} = 128kN, N_{2-3} = 50kN, N_{2-4} = -64,88kN, N_{2-5} = -55,93kN, N_{3-5} = 128kN, N_{6-8} = -41,33kN, N_{7-8} = -4,81kN, N_{7-9} = 64kN.

IV.28. N_{1-2}= -82,5kN, N_{1-3} = 18,33 kN, N_{2-3} = 50kN, N_{2-4} = -26,65kN, N_{2-5} = 11,77kN, N_{3-5} = 18,33kN, N_{6-8} = -35 kN, N_{7-8} = -11,78 kN, N_{7-9} = 43,34kN.

IV.29. N_{1-2}= -52kN, N_{1-3} = 31,12kN, N_{2-3} = -22kN, N_{2-4} = -74kN, N_{2-5} = 31,12kN, N_{3-5} = 52kN, N_{6-8} = -96kN, N_{7-8} = 82kN, N_{7-9} =38 kN.

IV.30. N_{1-2}= 18,07kN, N_{1-3} = -72,2kN, N_{2-3} = 27,93 kN, N_{2-4} = 21,72kN, N_{3-4} = -60 kN, N_{3-5} = -12,07kN, N_{6-8} = 22,67kN, N_{7-8} = 9,61 kN, N_{7-9} = -28kN.

IV.31. N_{1-2}= 1,33kN, N_{1-3} = -33,33kN, N_{2-3} = 19kN, N_{2-4} = 1,66kN, N_{3-4} = 13,33kN, N_{3-5} = -1kN, N_{6-8} = 33,99kN, N_{6-9} = -18,33 kN, N_{7-9} = -22,99kN.

IV.32. N_{1-2}= -60,4kN, N_{1-3} = 67kN, N_{2-3} = 54kN, N_{2-4} = -67kN, N_{3-4} = 51,45 kN, N_{3-5} = 44kN, N_{6-8} = -21kN, N_{7-8} = 51,45kN, N_{7-9} = -2kN.

IV.33. N_{1-2}= 50daN, N_{1-3} = -70,72daN, N_{2-3} = -74daN, N_{2-4} = -24daN, N_{2-5} = 104,67daN, N_{3-5} = 70daN, N_{6-8} = -72daN, N_{6-9} = -36,77daN, N_{7-9} = 78daN.

IV.34. N_{1-2}= -146,16daN, N_{1-3} = 63,33 daN, N_{2-3} = 60daN, N_{2-4} = -103,73daN, N_{2-5} = -42,43daN, N_{3-5} = 63,33daN, N_{6-8} = -82,03daN, N_{7-8} = -113,15daN, N_{7-9} = 153,34daN.

IV.35. N_{1-2}= -89,48daN, N_{1-3} = 140daN, N_{2-3} = 40daN, N_{2-4} = -44,74daN, N_{2-5} = -44,74daN, N_{3-5} = 140daN, N_{6-8} = -22,37daN, N_{6-9} = -12daN, N_{7-9} = 86,67daN.

IV.36. N_{1-2}= -45,83kN, N_{1-3} = -13,33kN, N_{2-3} = 40kN, N_{2-4} = 12,5kN, N_{2-5} = -33,33 kN, N_{3-5} = -13,33kN, N_{6-8} = 4,16kN, N_{6-9} = -10,95kN, N_{7-9} = -48,88kN.

IV.37. N_{1-2}= -259,49daN, N_{1-3} = 186daN, N_{2-3} = 50daN, N_{2-4} = -180,37daN, N_{2-5} = -79,11daN, N_{3-5} = 186daN, N_{6-8} = -178daN, N_{6-9} = 45,26daN, N_{7-9} = 86daN.

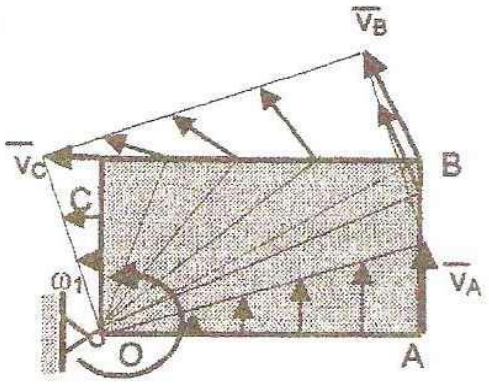
IV.38. N_{1-2}= 130daN, N_{1-3} = -232,5daN, N_{2-4} = 330daN, N_{1-4} = -162,5daN, N_{3-4} = 130daN, N_{4-6} = 232,5daN, N_{6-8} = 147,78daN, N_{5-8} = -15,06daN, N_{5-7} = -123,59daN.

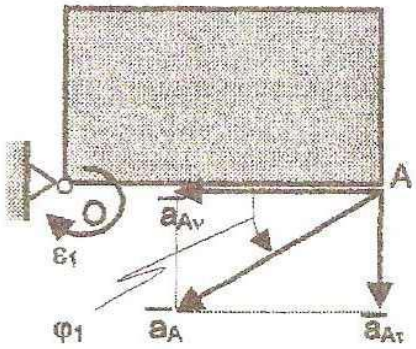
IV.39. N_{1-2}= -63,88kN, N_{1-3} = 78,33kN, N_{2-3} = 51,1kN, N_{2-4} = -78,33kN, N_{3-4} = -11,12kN, N_{3-5} = 71,65kN, N_{6-8} = -58,33kN, N_{6-9} = -11,11kN, N_{7-9} = 65kN.

IV.40. N_{1-2}= 40daN, N_{1-3} = 0, N_{2-3} = -48,07daN, N_{2-4} = -3,36daN, N_{3-4} = -20daN, N_{4-5} = -3,36daN, N_{6-8} = 18,33daN, N_{7-8} = 19,53daN, N_{7-9} = -29,17daN.

CAPITOLUL V. CINEMATICA SOLIDULUI RIGID
V.1. MIŞCAREA DE ROTAŢIE PLANĂ

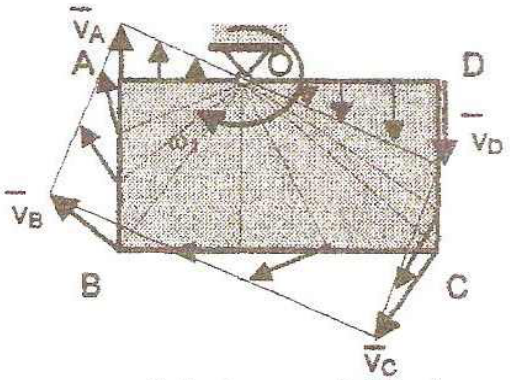
V.1.1.

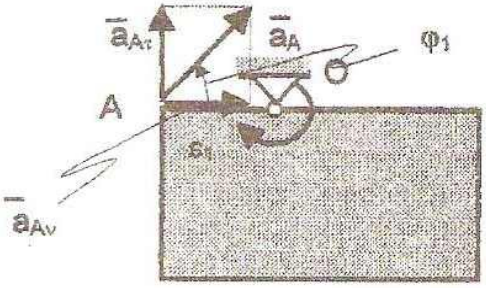




$v_A = 150cm/s$, $v_B = 159cm/s$, $v_C = 60cm/s$,
$a_{A\tau} = 300cm/s^2$, $a_{A\nu} = 450cm/s^2$,$a_A = 541cm/s^2$, $\varphi_1 = 33,42^\circ$

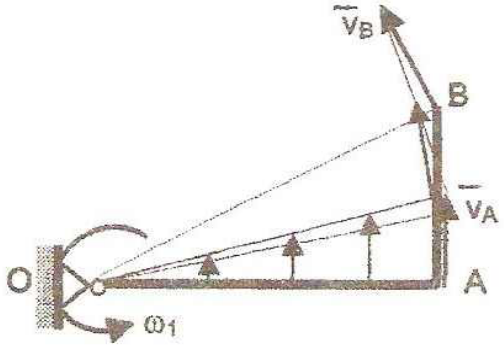
V.1.2.

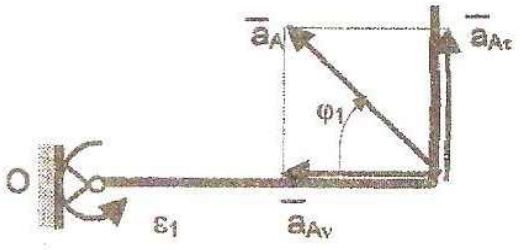




$v_A = 0,4m/s$, $v_B = 0,72m/s$, $v_C = 0,848m/s$, $v_D = 0,6\ m/s$
$a_{A\tau} = 1\ m/s^2$, $a_{A\nu} = 0,8\ m/s^2$,$a_A = 1,28\ m/s^2$, $\varphi_1 = 51,34^\circ$

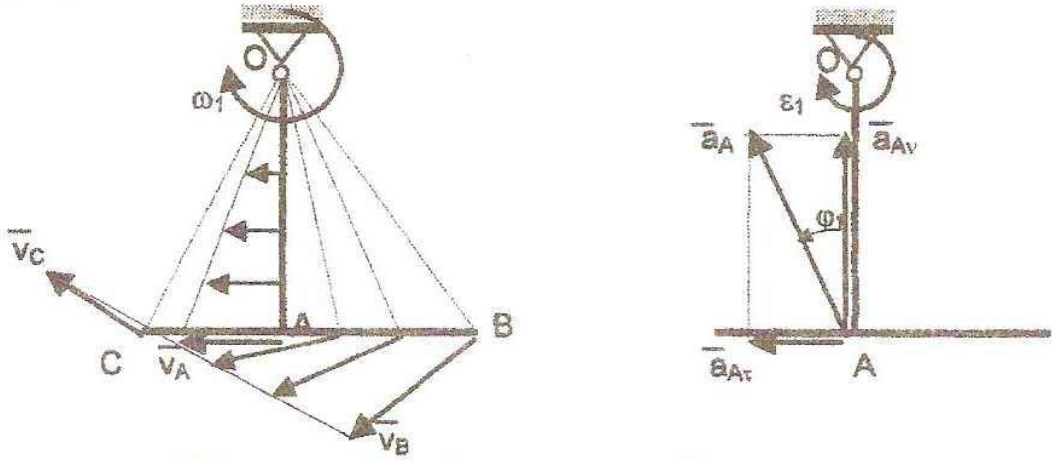
V.1.3.





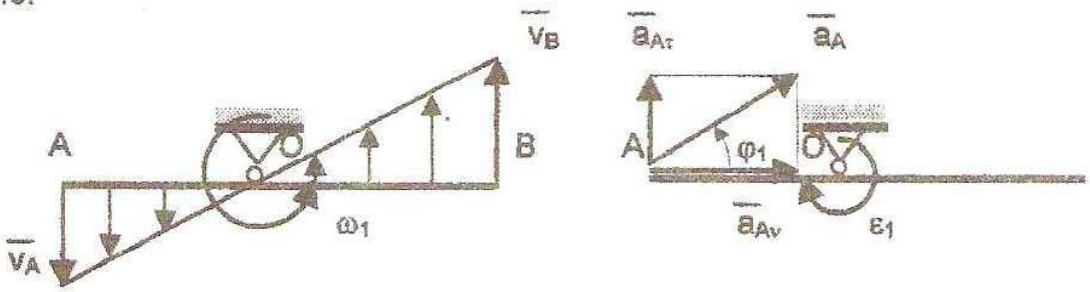
v_A = 1,2m/s, v_B = 1,44m/s,
$a_{A\tau}$ = 2,4 m/s^2, a_{Av} = 2,4 m/s^2 ,a_A = 3,39 m/s^2 , φ_1 = 45°

V.1.4.



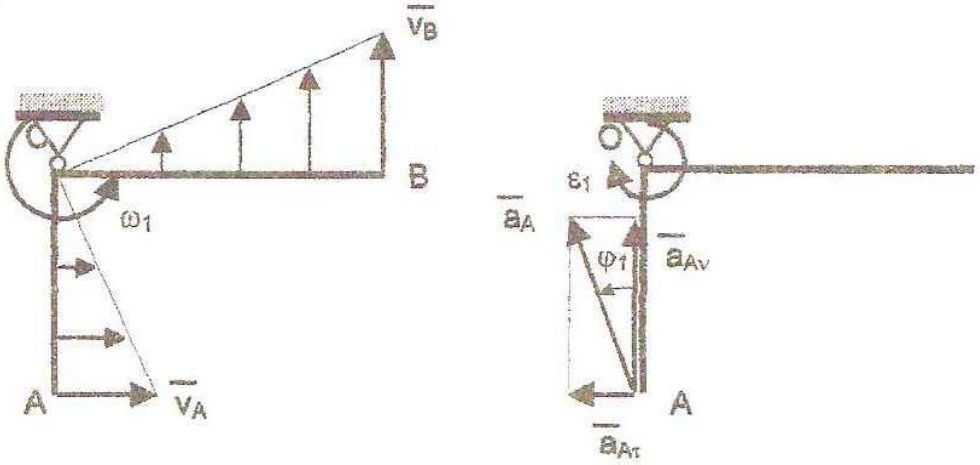
v_A = 225cm/s, v_B = 270,41cm/s, v_C = 246,22 cm/s,
$a_{A\tau}$ = 270cm/s^2, a_{Av} = 1125cm/s^2, a_A = 1156cm/s^2, φ_1 = 13,49°

V.1.5.



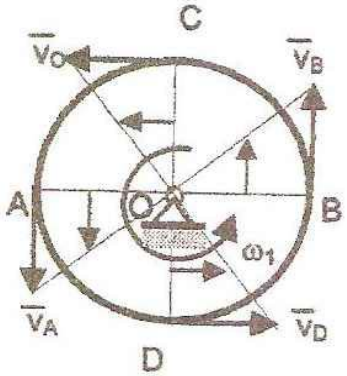
v_A = 1,8m/s, v_B = 2,4 m/s,
$a_{A\tau}$ = 2,4 m/s^2, a_{Av} = 5,4 m/s^2, a_A = 5,9 m/s^2, φ_1 = 23,96°

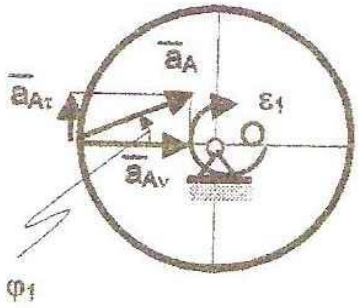
V.1.6.



$v_A = 2$ m/s, $v_B = 2{,}8$ m/s,
$a_{A\tau} = 2$ m/s^2, $a_{Av} = 8$ m/s^2, $a_A = 8{,}24$m/s^2, $\varphi_1 = 14{,}03°$

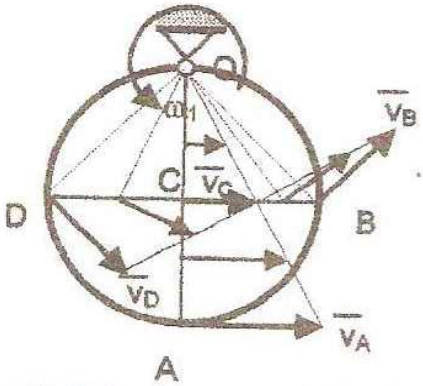
V.1.7.

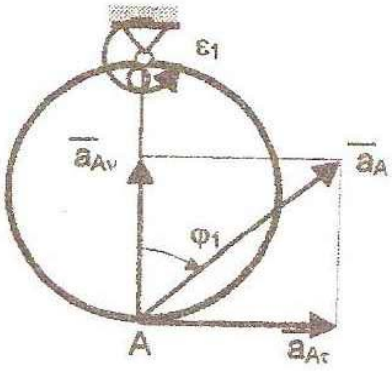




$v_A = v_B = v_C = v_D = 200$cm/s,
$a_{A\tau} = 250$cm/s^2, $a_{Av} = 800$cm/s^2, $a_A = 838$cm/s^2, $\varphi_1 = 17{,}35°$

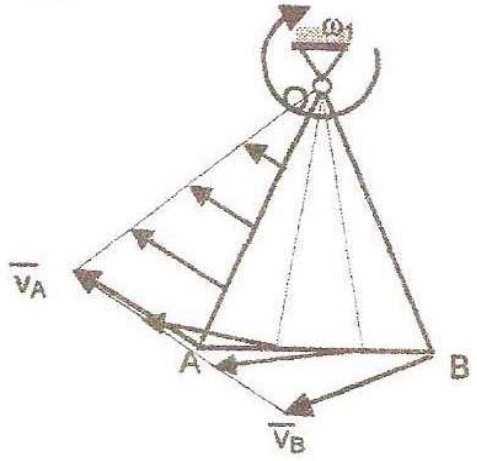
V.1.8.

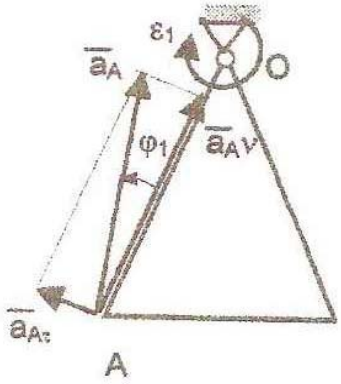




$v_A = 0{,}8$m/s, $v_B = v_D = 0{,}56$ m/s, $v_C = 0{,}4$m/s,
$a_{A\tau} = 2$ m/s^2, $a_{Av} = 1{,}6$m/s^2, $a_A = 2{,}56$ m/s^2, $\varphi_1 = 51{,}34°$

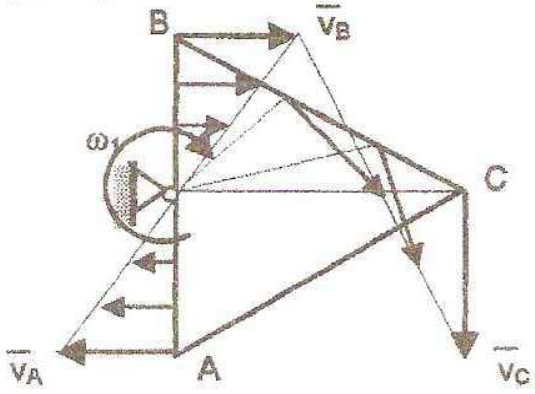
V.1.9.

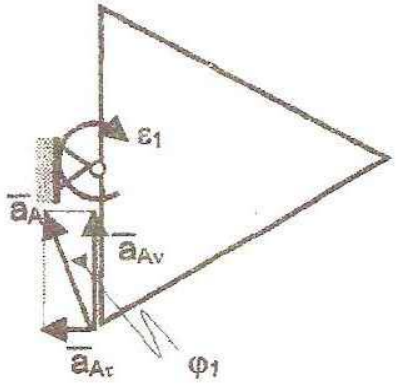




$v_A = v_B = 300cm/s$,
$a_{A\tau} = 180cm/s^2$, $a_{Av} = 1500cm/s^2$, $a_A = 1511cm/s^2$, $\varphi_1 = 6,84°$

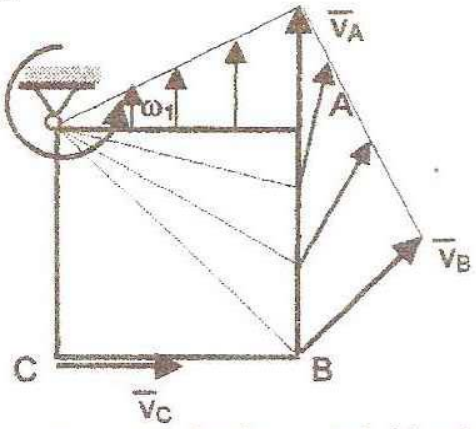
V.1.10.

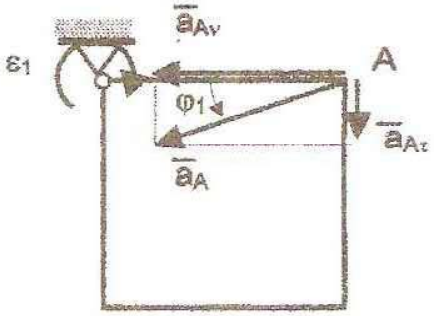




$v_A = v_B = 0,6m/s$, $v_C = 1,04$ m/s,
$a_{A\tau} = 0,6$ m/s^2, $a_{Av} = 2,4m/s^2$, $a_A = 2,47m/s^2$, $\varphi_1 = 14,03°$

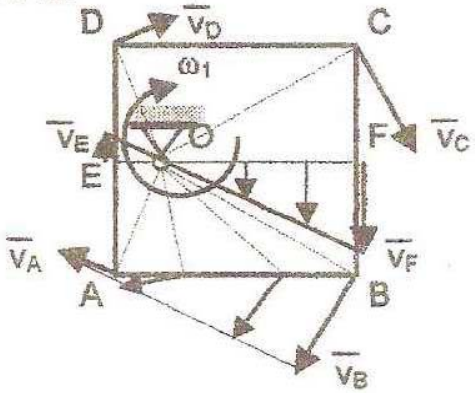
V.1.11.

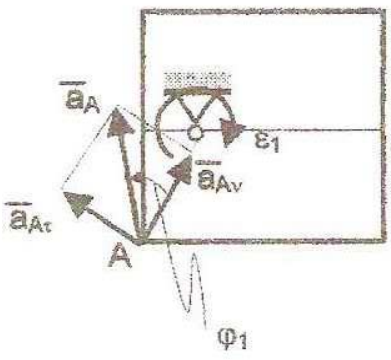




$v_A = v_C = 1,2$ m/s, $v_B = 1,69$ m/s,
$a_{A\tau} = 0,6m/s^2$, $a_{Av} = 2,4m/s^2$, $a_A = 2,47m/s^2$, $\varphi_1 = 14,03°$

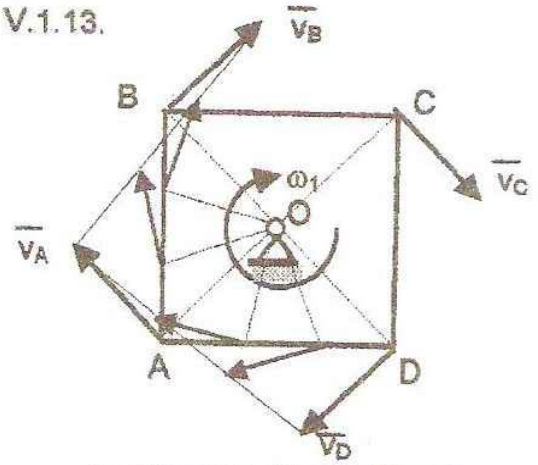
V.1.12.

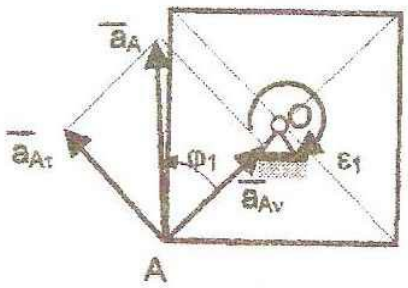




$v_A = v_D = 67$ cm/s, $v_B = v_C = 108$ cm/s, $v_F = 90$ cm/s, $v_E = 30$ cm/s,
$a_{A\tau} = 134 cm/s^2$, $a_{Av} = 201 cm/s^2$, $a_A = 241 cm/s^2$, $\varphi_1 = 33{,}42°$

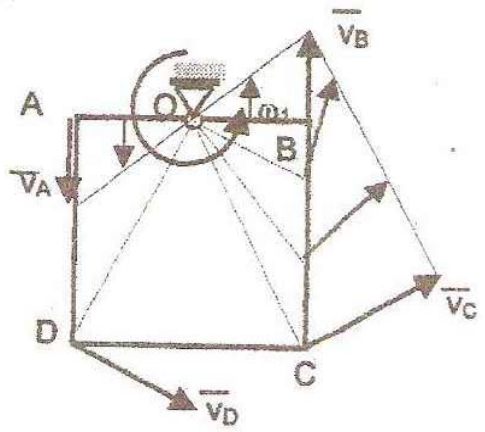
V.1.13.

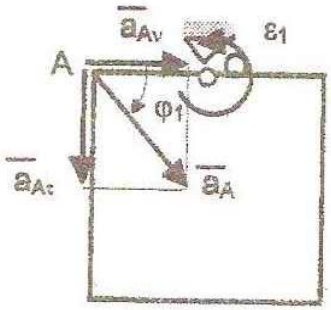




$v_A = v_B = v_C = v_D = 42$cm/s,
$a_{A\tau} = 70 cm/s^2$, $a_{Av} = 63 cm/s^2$, $a_A = 94 cm/s^2$, $\varphi_1 = 48°$

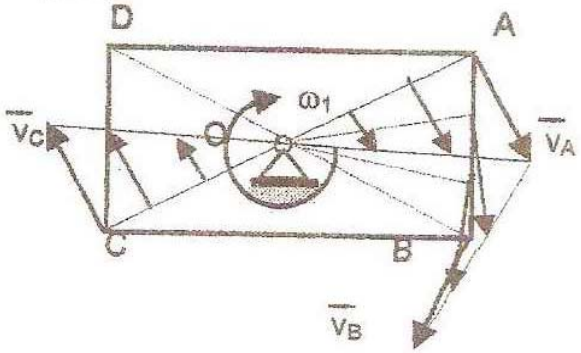
V.1.14.

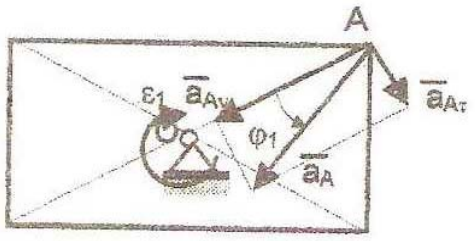




$v_A = v_B = 1$ m/s, $v_C = v_D = 2{,}23$ m/s,
$a_{A\tau} = 4$ m/s^2, $a_{Av} = 5$ m/s^2, $a_A = 6{,}4$m/s^2, $\varphi_1 = 38{,}65°$

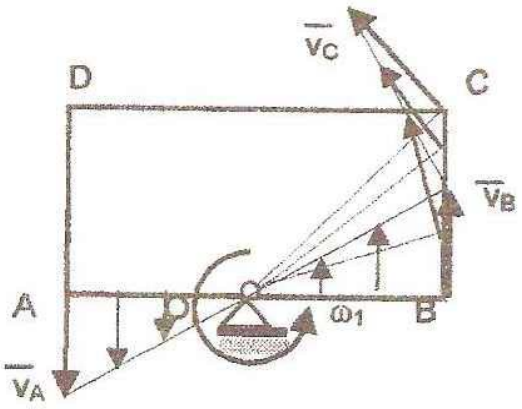
V.1.15.

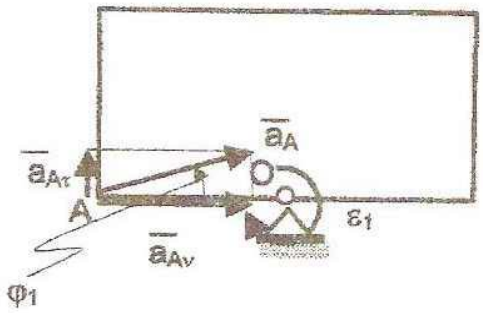




$v_A = v_B = v_C = v_D = 141 cm/s$,
$a_{A\tau} = 100 cm/s^2$, $a_{Av} = 493 cm/s^2$, $a_A = 503 cm/s^2$, $\varphi_1 = 11,46°$

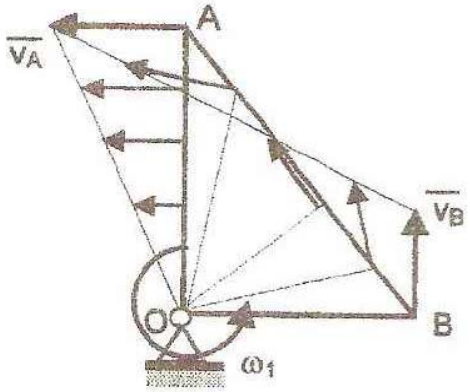
V.1.16.

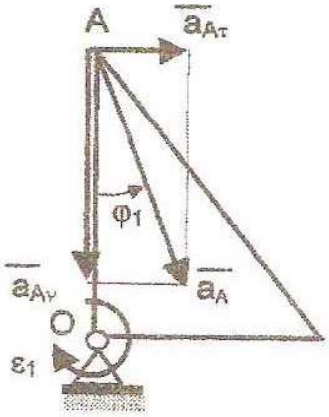




$v_A = v_B = 120$ cm/s, $v_C = v_D = 200 cm/s$,
$a_{A\tau} = 60 cm/s^2$, $a_{Av} = 480 cm/s^2$, $a_A = 483 cm/s^2$, $\varphi_1 = 7,12°$

V.1.17.

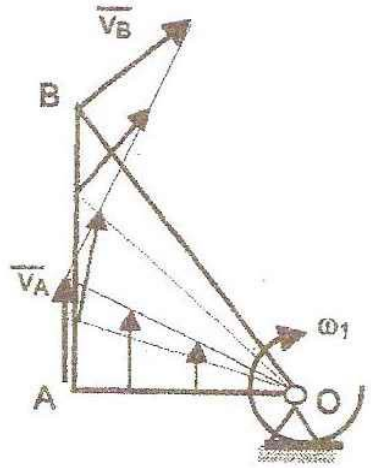


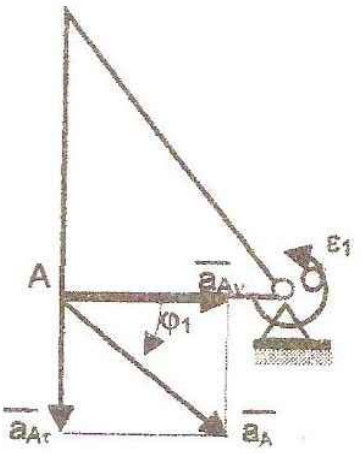


$v_A = 240 cm/s$, $v_B = 180 cm/s$,
$a_{A\tau} = 200 cm/s^2$, $a_{Av} = 720 cm/s^2$, $a_A = 747 cm/s^2$, $\varphi_1 = 15,52°$

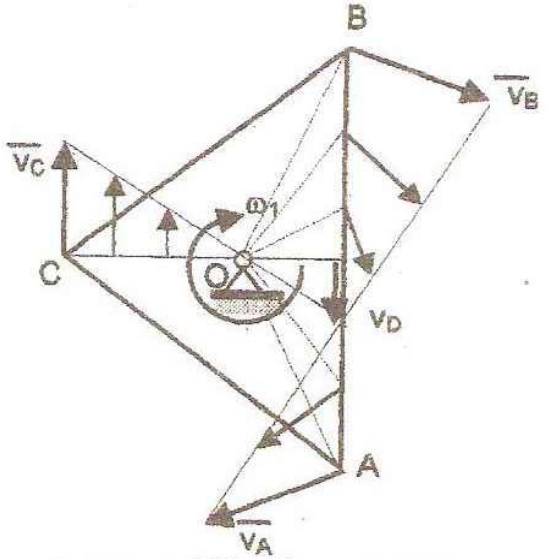
V.1.18.

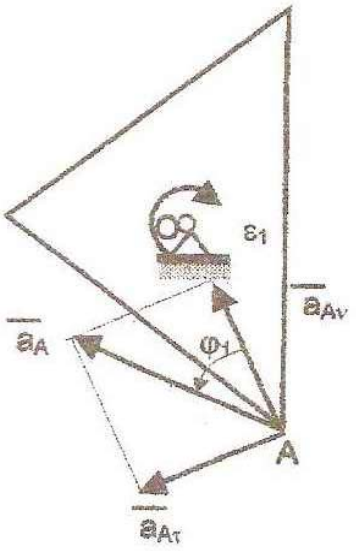
$v_A = 0,9 m/s$, $v_B = 1,5 m/s$,
$a_{A\tau} = 2,4 m/s^2$, $a_{Av} = 2,7 m/s^2$, $a_A = 3,61 m/s^2$, $\varphi_1 = 41,63°$





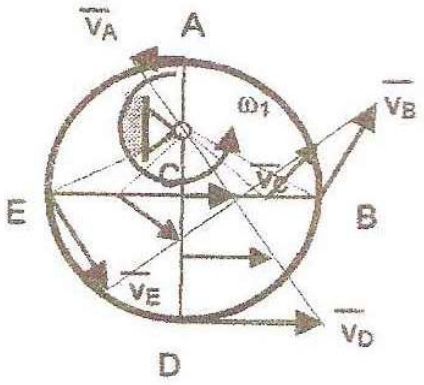
V.1.19.

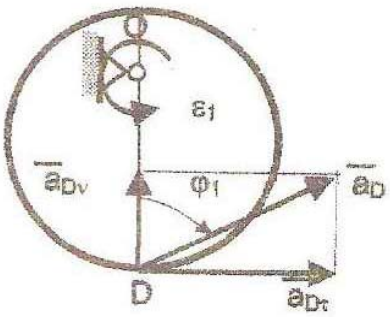




$v_A = v_B = 160cm/s$, $v_C = 135cm/s$,
$a_{A\tau} = 800cm/s^2$, $a_{Av} = 800cm/s^2$, $a_A = 1131cm/s^2$, $\varphi_1 = 45°$

V.1.20.

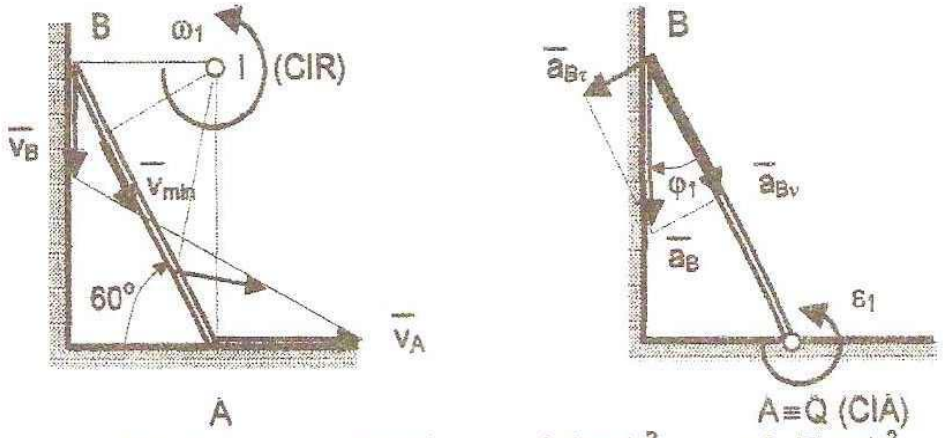




$v_A = v_C = 0{,}6$ m/s, $v_D = 1{,}8$m/s, $v_B = v_E = 1{,}34$ m/s,
$a_{A\tau} = 7{,}2$m/s^2, $a_{Av} = 3{,}6$m/s^2, $a_A = 8{,}04$m/s^2, $\varphi_1 = 63{,}43^\circ$

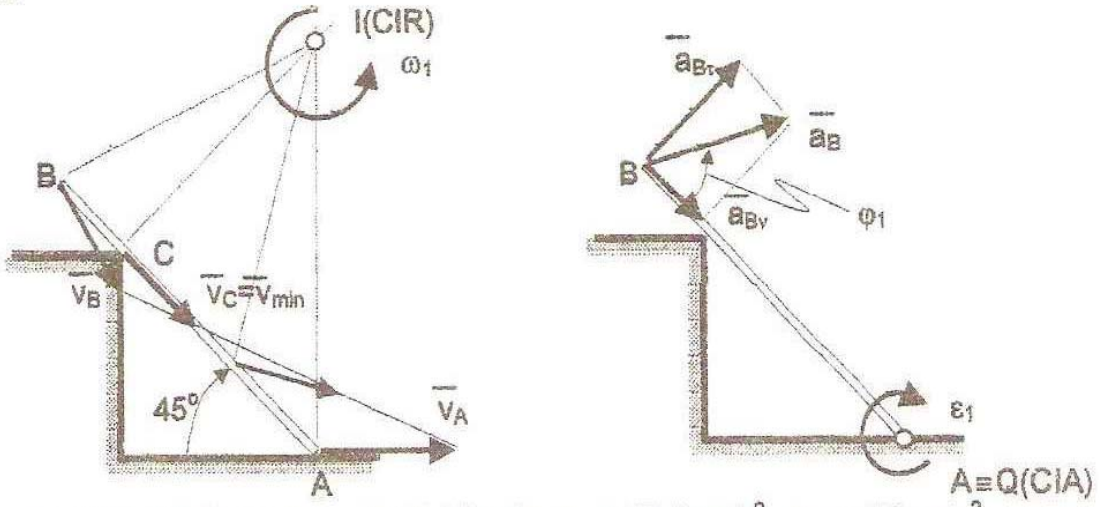
V.2. MIŞCAREA PLAN PARALELĂ (ÎN PLAN)

V.2.1.



$v_B = 5{,}78$cm/s, $v_{min} = 4{,}93$cm/s, $a_{B\tau} = 3{,}8$cm/s^2 , $a_{Bv} = 6{,}49$cm/s^2 ,
$a_B = 7{,}52$cm/s^2 , $\varphi_1 = 30^\circ$.

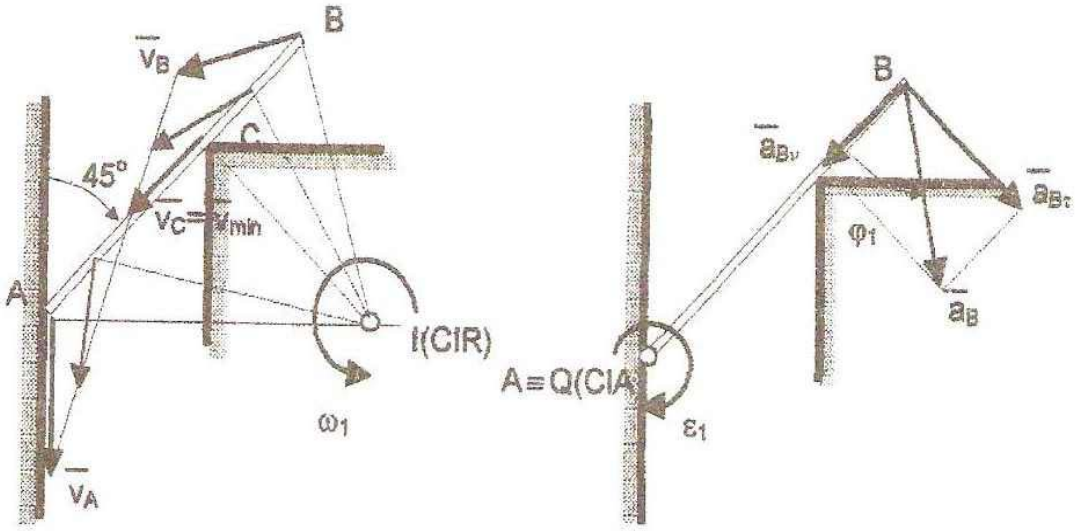
V.2.2.



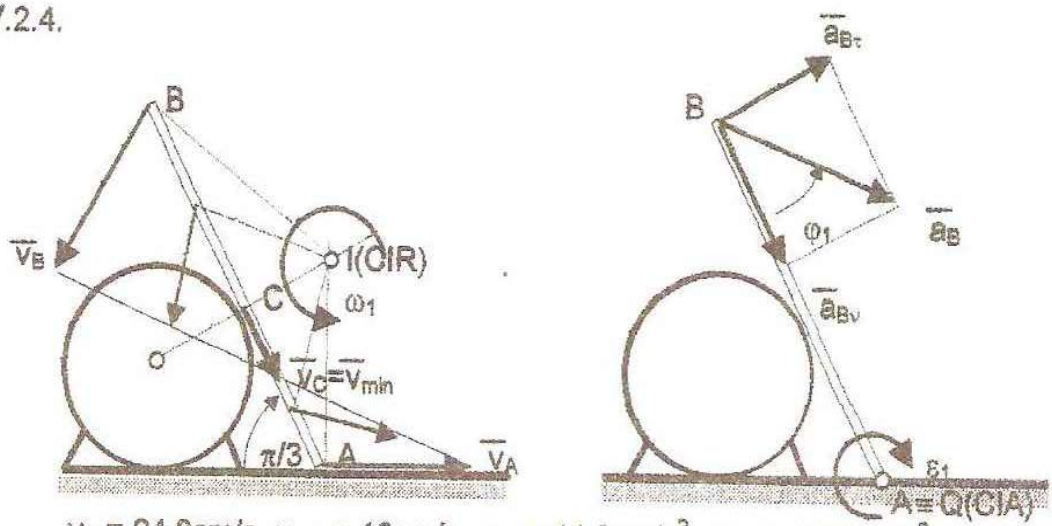
$v_B = 15{,}3$cm/s, $v_C = v_{min} = 14{,}14$cm/s, $a_{B\tau} = 39{,}8$cm/s^2 , $a_{Bv} = 20$cm/s^2 ,
$a_B = 44{,}54$cm/s^2 , $\varphi_1 = 63{,}32^\circ$.

V.2.3.

$v_B = 0{,}38$m/s, $v_{min} = 0{,}35$m/s, $a_{B\tau} = 0{,}83$m/s^2 , $a_{Bv} = 0{,}43$m/s^2 , $a_B = 0{,}93$m/s^2 ,
$\varphi_1 = 62{,}61^\circ$.

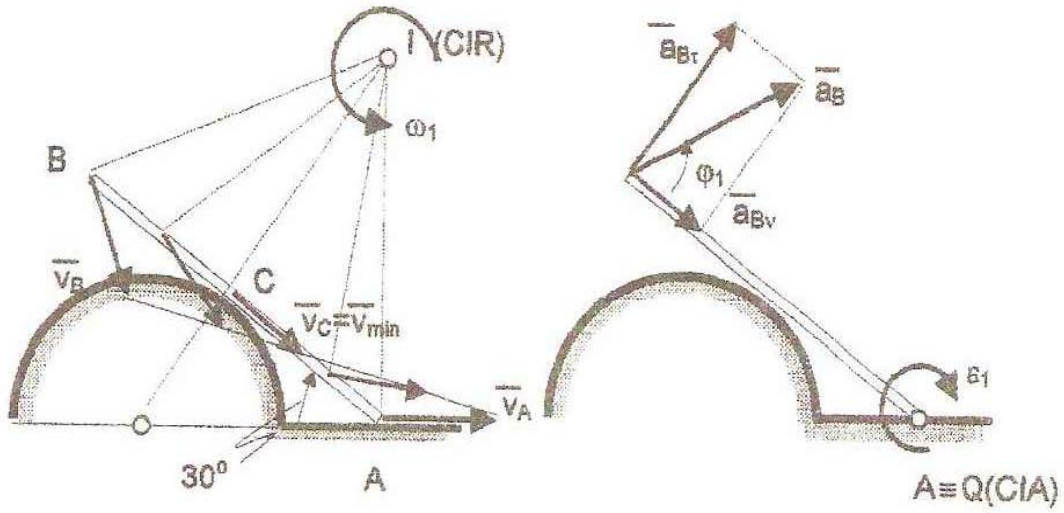


V.2.4.



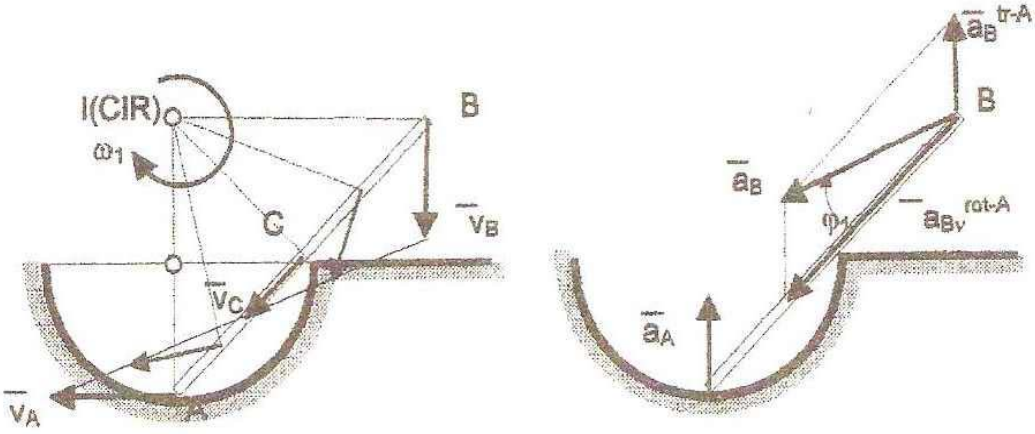
$v_B = 24{,}8\text{cm/s}$, $v_{min} = 10\text{cm/s}$, $a_{B\tau} = 11{,}6\text{cm/s}^2$, $a_{Bv} = 13{,}3\text{cm/s}^2$, $a_B = 17{,}7\text{cm/s}^2$, $\varphi_1 = 41°$.

V.2.5.



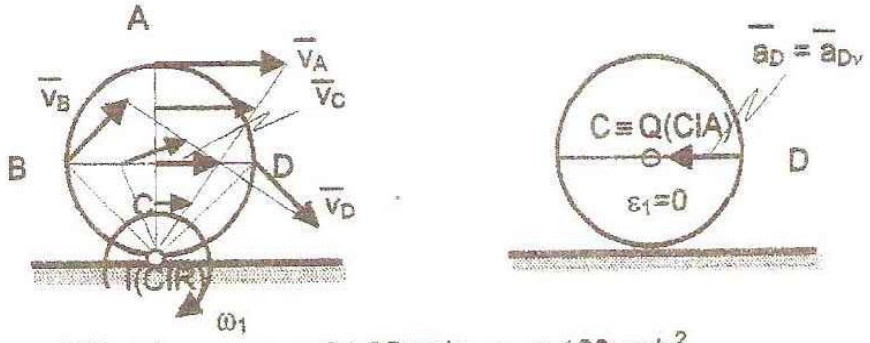
$v_B = 0{,}37 m/s$, $v_{min} = 0{,}34 m/s$, $a_{B\tau} = 0{,}65 m/s^2$, $a_{Bv} = 0{,}16 m/s^2$, $a_B = 0{,}67 m/s^2$, $\varphi_1 = 76{,}17°$.

V.2.6.



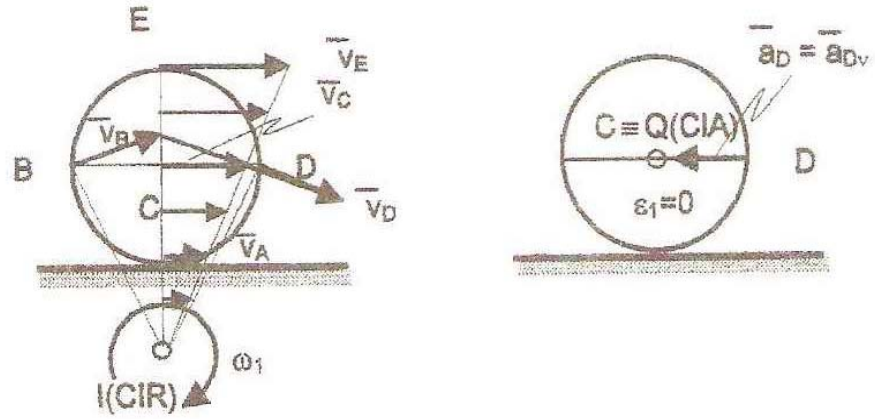
$v_B = 53 cm/s$, $v_{min} = 35 cm/s$, $a_B^{tr-A} = 31{,}3\ cm/s^2$, $a_{Bv}^{rot-A} = 93{,}8 cm/s^2$, $a_B = 75 cm/s^2$, $\varphi_1 = 17{,}45°$

V.2.7.



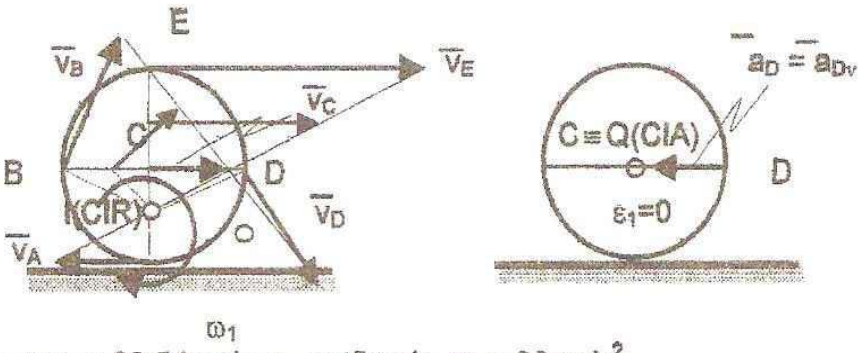
$v_A = 120 cm/s$, $v_B = v_D = 84{,}85 cm/s$, $a_D = 120 cm/s^2$.

V.2.8.



$v_B = v_D = 1{,}8 m/s$, $v_E = 2{,}5 m/s$, $a_D = 2 m/s^2$.

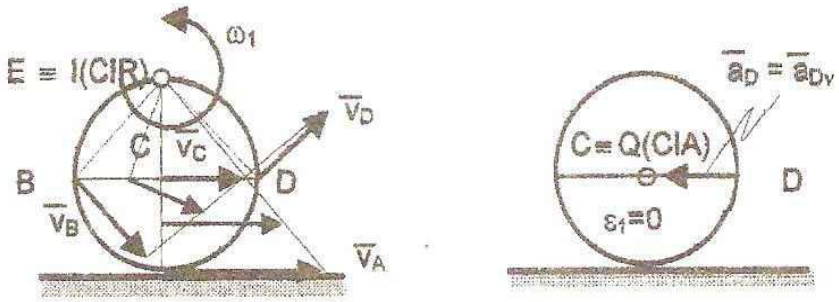
V.2.9



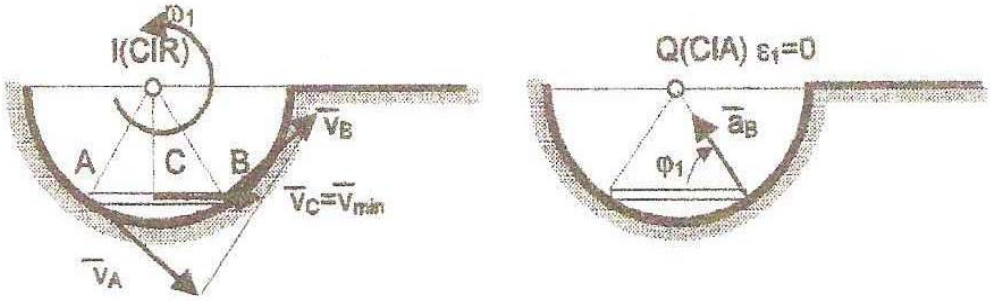
$v_B = v_D = 33{,}54 cm/s$, $v_E = 45 cm/s$, $a_D = 90 cm/s^2$.

V.2.10.

$v_B = v_B = 42{,}42 cm/s$, $v_E = 0$, $a_B = 45 cm/s^2$.



V.2.11.



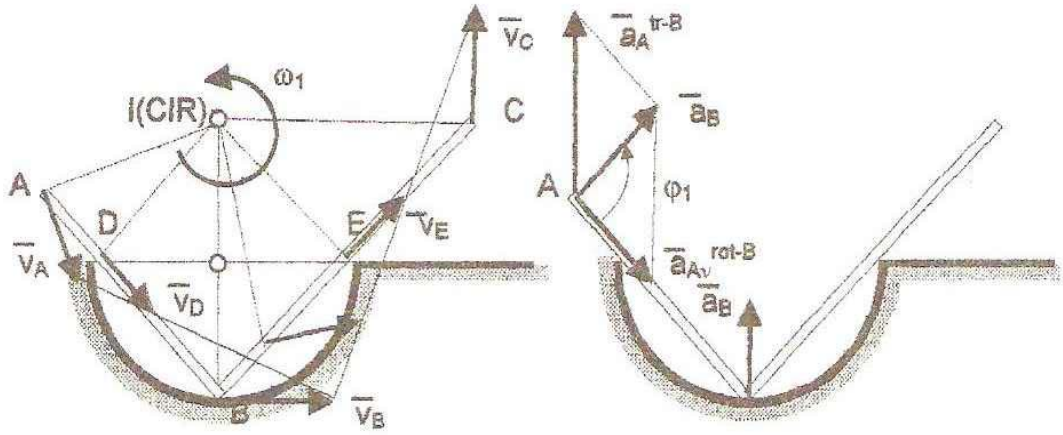
$v_B = 60 cm/s$, $v_{min} = 51{,}96 cm/s$, $a_B = 180 cm/s^2$, $\varphi_1 = 60^\circ$.

Întrucât CIR şi CIA se confundă şi se găsesc într-un punct fix această mişcare este de fapt o rotaţie cu centrul de rotaţie în O (centrul cercului).

V.2.12.

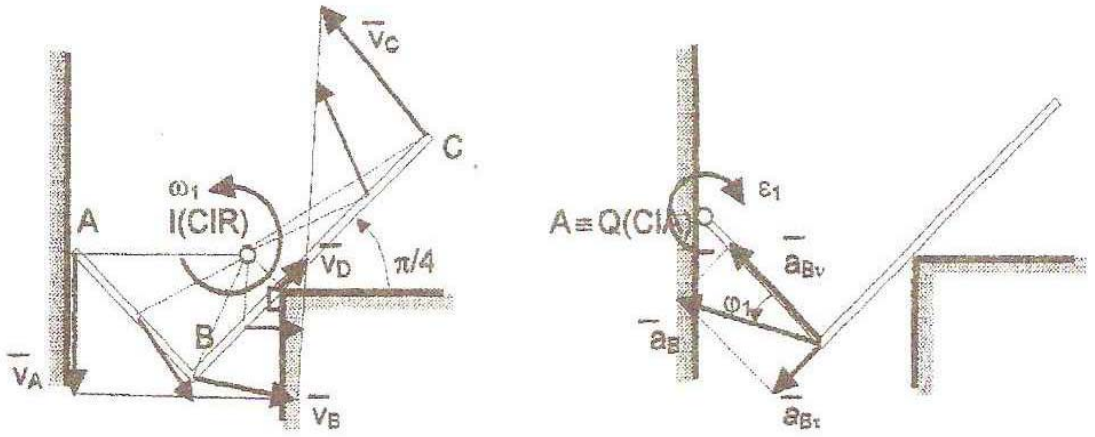
$v_A = 0{,}245 m/s$, $v_D = v_E = 0{,}14 m/s$, $v_C = 0{,}33 m/s$, $a_A^{tr-B} = 0{,}1 m/s^2$,

$a_{Av}^{rot-B} = 0{,}05 m/s^2$ $a_A = 0{,}074 m/s^2$, $\varphi_1 = 73°$.

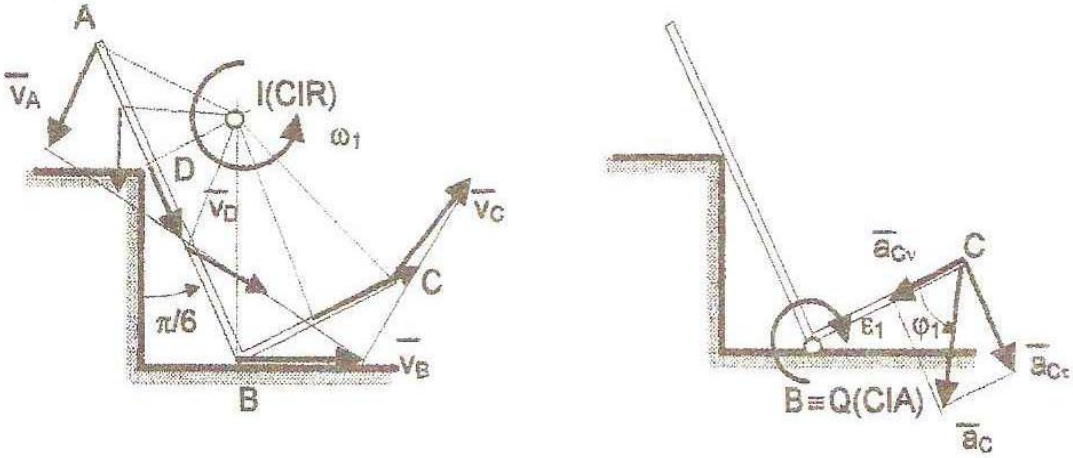


V.2.13.

$v_B = 6{,}17 m/s$, $v_C = 14{,}64 m/s$, $v_C = 5{,}17 m/s$, $a_{Bt} = 48{,}33 m/s^2$, $a_{Bv} = 123{,}72 m/s^2$, $a_B = 132{,}82 m/s^2$, $\varphi_1 = 21{,}33°$

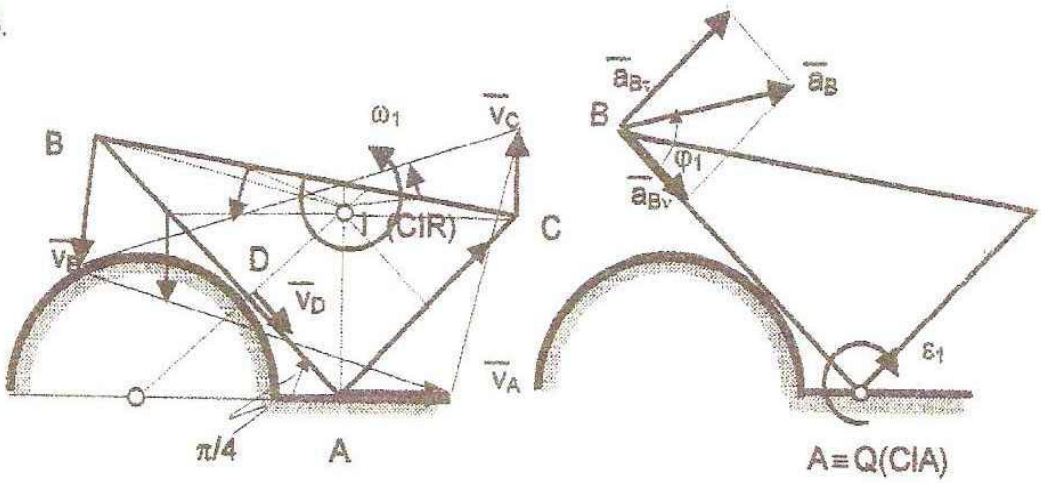


V.2.14.



$v_A = 0{,}65\text{m/s}$, $v_C = 0{,}74\text{m/s}$, $v_D = 0{,}38\text{m/s}$, $a_{C\tau} = 0{,}83\text{m/s}^2$, $a_{Cv} = 0{,}75\text{m/s}^2$, $a_C = 1{,}12\text{m/s}^2$, $\varphi_1 = 47{,}89^\circ$.

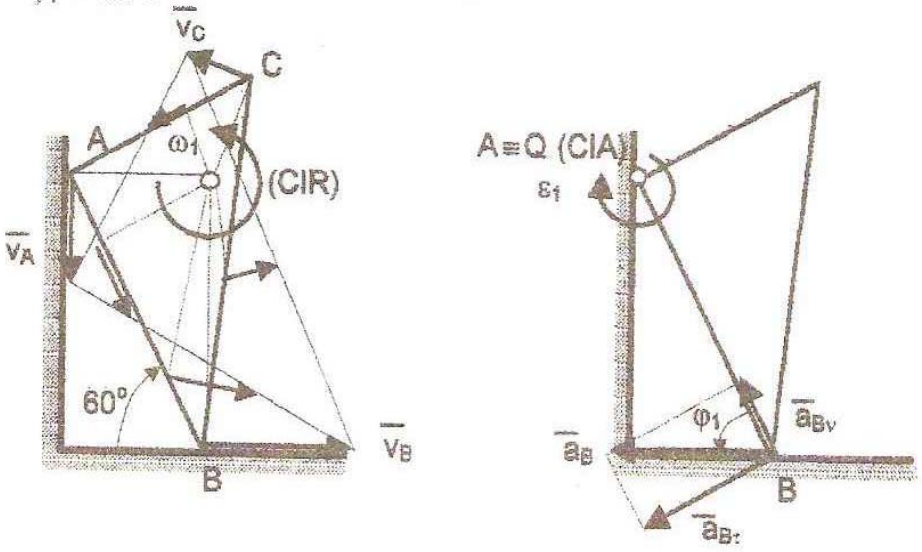
V.2.15.



$v_B = 0{,}47\text{m/s}$, $v_C = 0{,}41\text{m/s}$, $v_D = 0{,}21\text{m/s}$, $a_{B\tau} = 0{,}67\text{m/s}^2$, $a_{Bv} = 0{,}22\text{m/s}^2$, $a_B = 0{,}7\text{m/s}^2$, $\varphi_1 = 71{,}82^\circ$.

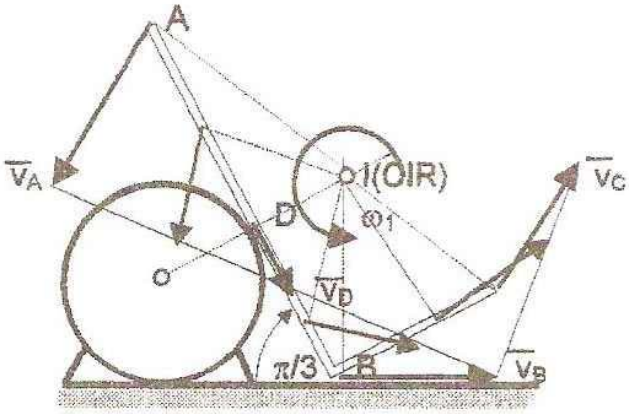
V.2.16

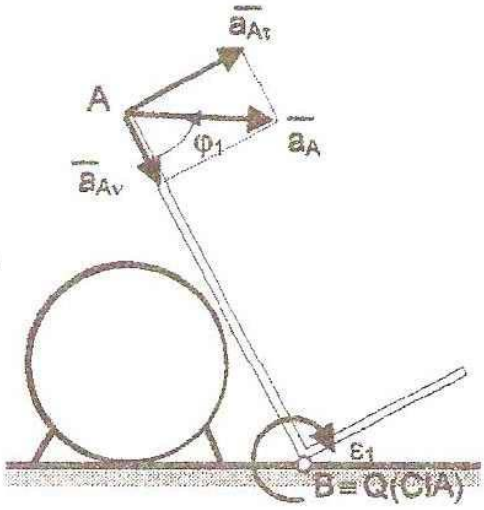
$v_B = 51\text{cm/s}$, $v_C = 24\text{cm/s}$, $a_{B\tau} = 155\text{cm/s}^2$, $a_{Bv} = 90\text{cm/s}^2$, $a_B = 179\text{cm/s}^2$, $\varphi_1 = 60^\circ$.



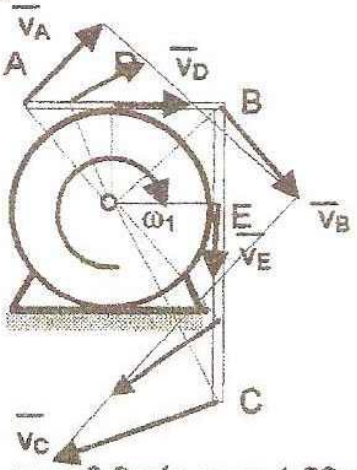
V.2.17.

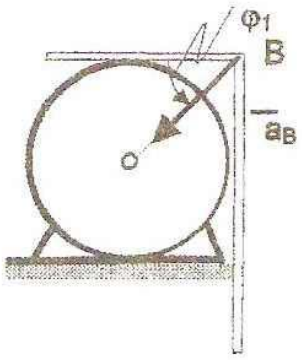
$v_A = 0{,}99\text{m/s}$, $v_C = 0{,}693\text{m/s}$, $v_D = 0{,}4\text{m/s}$, $a_{A\tau} = 4{,}64\text{m/s}^2$, $a_{Av} = 1{,}6\text{m/s}^2$, $a_A = 4{,}9\text{m/s}^2$, $\varphi_1 = 71^\circ$.





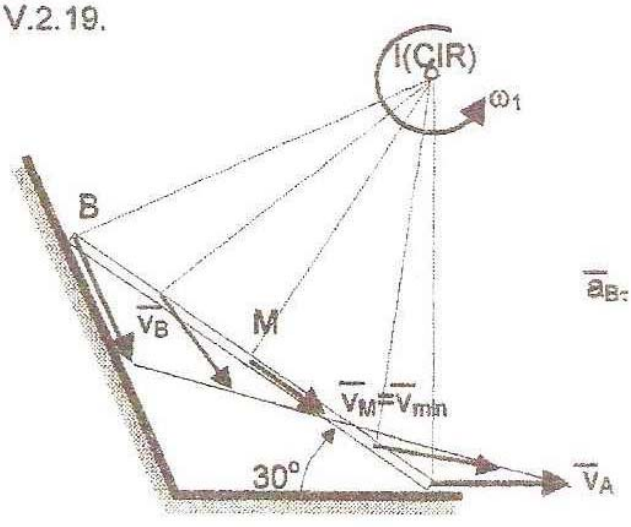
V.2.18.

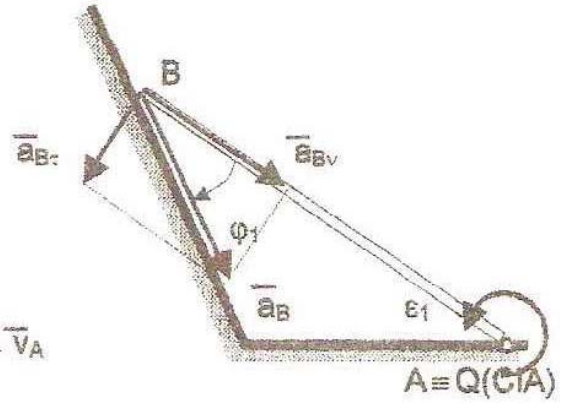




$v_A = 0{,}8\text{m/s}$, $v_C = 1{,}26\text{m/s}$, $v_D = v_E = 0{,}57\text{m/s}$, $a_B = 0{,}9\text{m/s}^2$, $\varphi_1 = 45°$.

V.2.19.

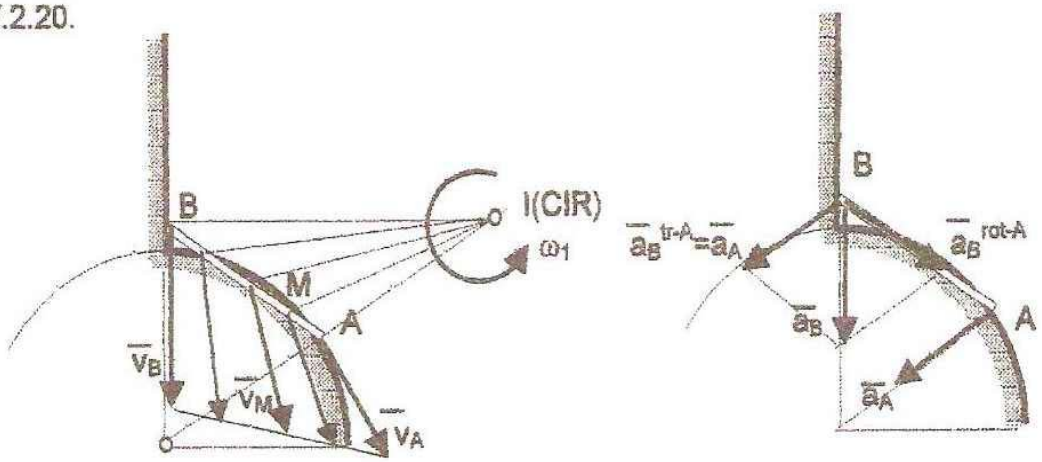




A

$v_B = 0,5m/s$, $v_{min} = 0,44m/s$, $a_{Bt} = 0,12m/s^2$, $a_{Bv} = 0,21m/s^2$, $a_B = 0,242m/s^2$, $\varphi_1 = 30^\circ$.

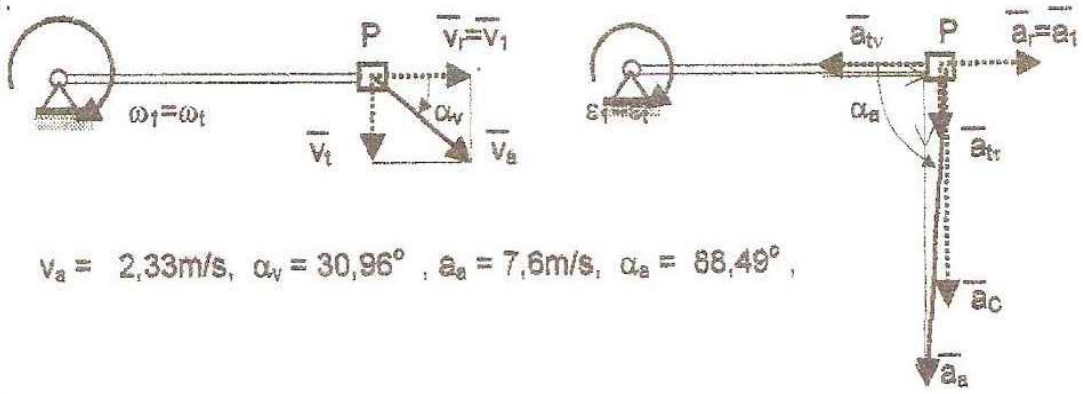
V.2.20.



$v_B = 3,47m/s$, $v_M = 1,45m/s$, $a_B^{tr-A} = 3,33m/s^2$, $a_B^{rot-A} = 3,33m/s^2$, $a_B = 3,33m/s^2$.

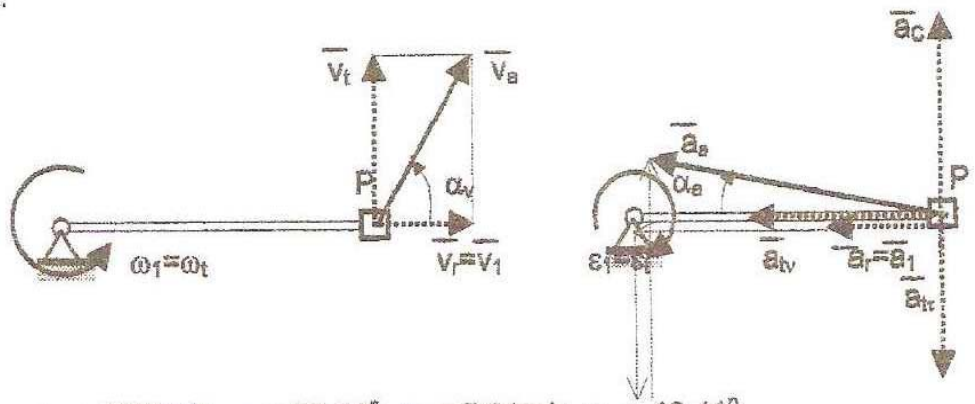
CAPITOLUL VI. MIŞCAREA RELATIVĂ A PUNCTULUI

VI.1.



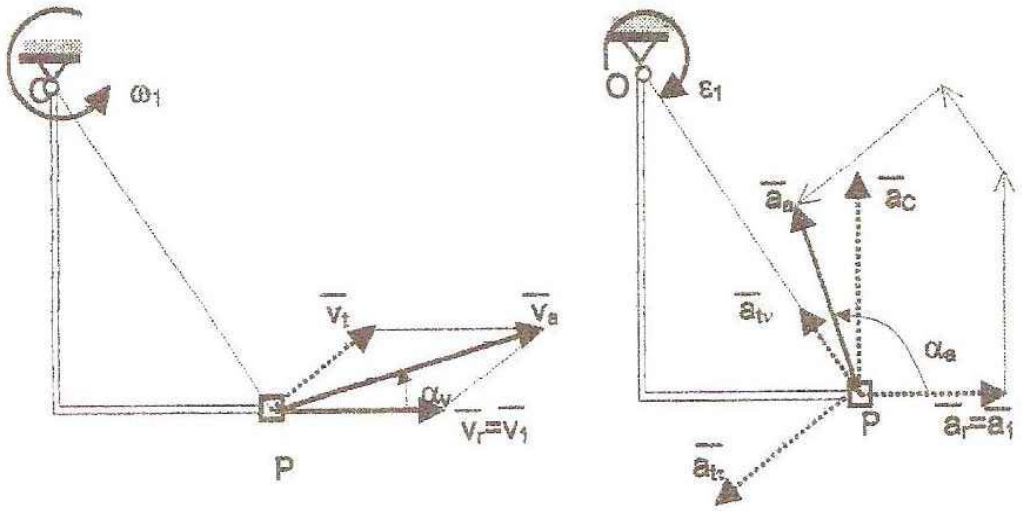
$v_a = 2,33m/s$, $\alpha_v = 30,96^\circ$, $a_a = 7,6m/s$, $\alpha_a = 88,49^\circ$,

VI.2.



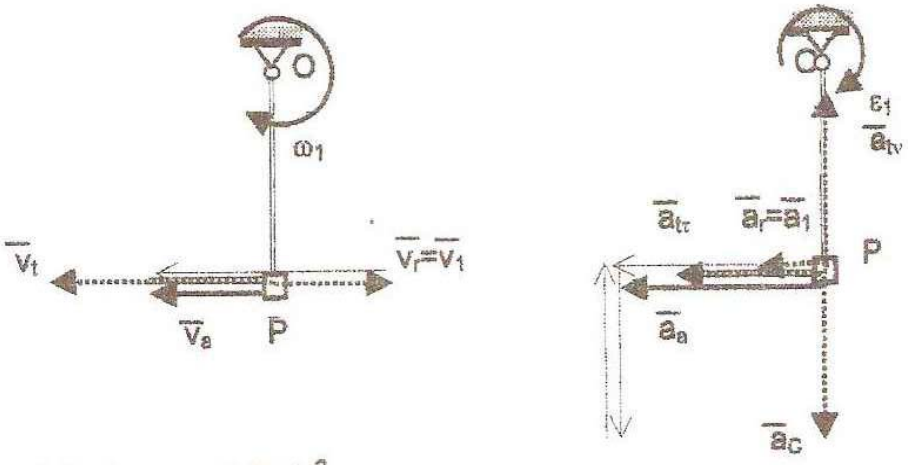
$v_a = 2,83m/s$, $\alpha_v = 57,99^\circ$, $a_a = 7,21m/s$, $\alpha_a = 19,41^\circ$

VI.3.



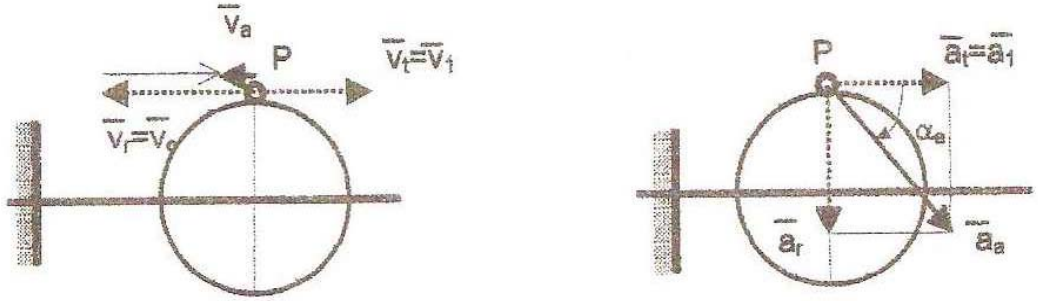
$v_a = 1044 cm/s$; $\alpha_v = 16,7°$; $a_a = 1044 cm/s^2$; $\alpha_v = 94,95°$

VI.4.



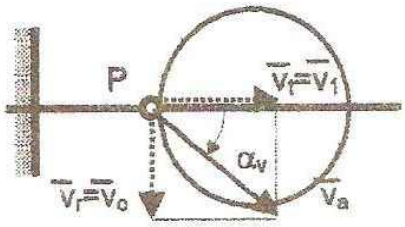
$v_a = 0.6 m/s$; $a_a = 2,6 m/s^2$

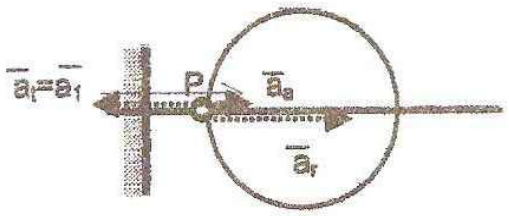
VI.5.



$v_a = 10$ cm/s, $a_a = 100$ cm/s^2, $\alpha_a =$ 53,13°.

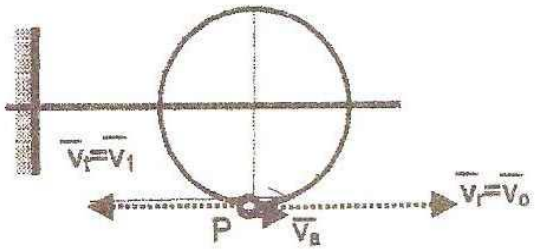
VI.6.

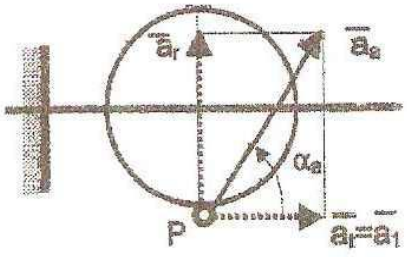




v_a = 064m/s; α_v =38,62°; a_a = 0,03m/s^2 .

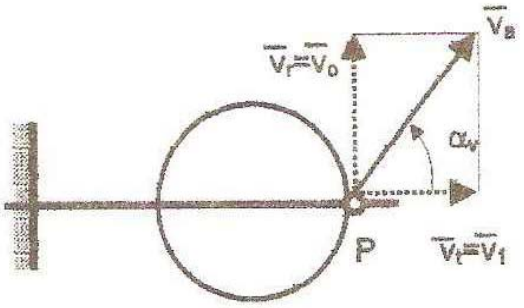
VI.7.

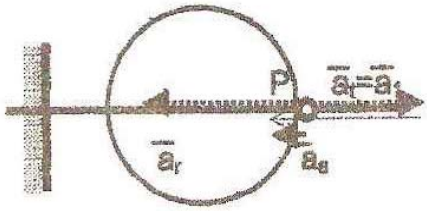




v_a = 10 cm/s, a_a = 170 cm/s^2, α_a = 58,03°

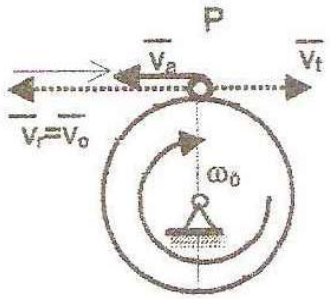
VI.8.

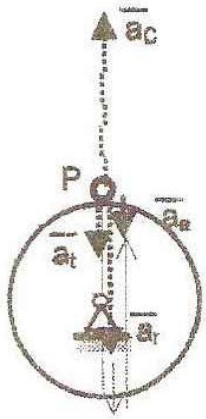




v_a = 2,5m/s; α_v = 53,13° ; a_a = 0,33m/s^2

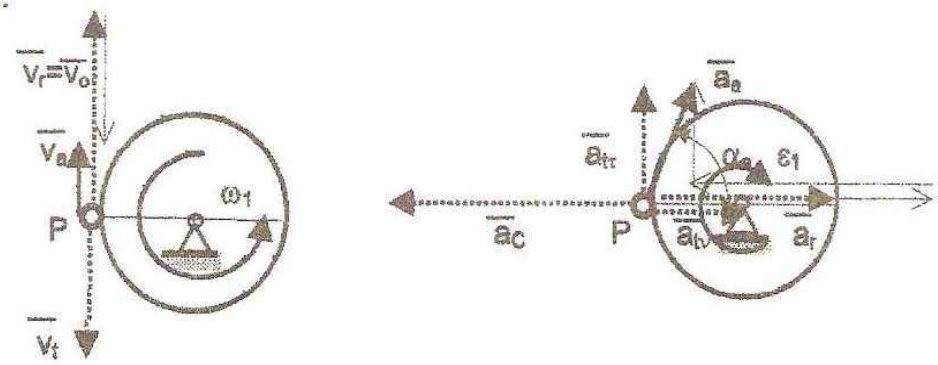
VI.9.





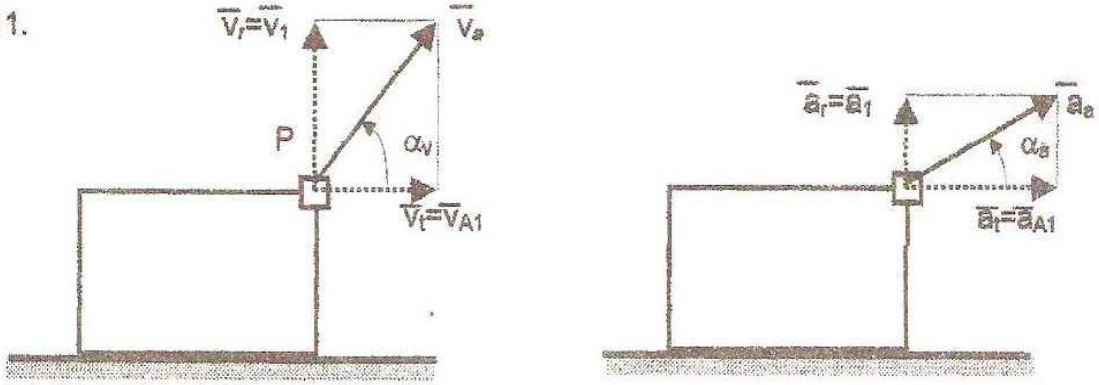
$v_a = 1{,}2 m/s;\ a_a = 2{,}4 m/s^2$

VI.10.



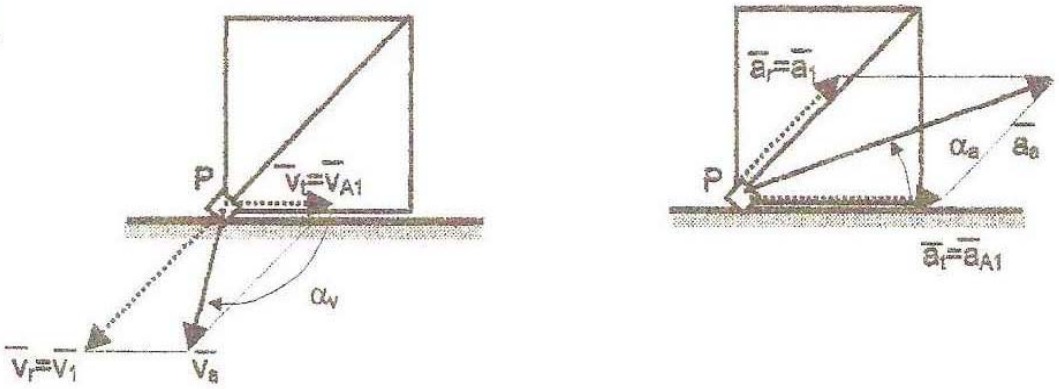
$v_a = 0{,}3 m/s,\ a_a = 1{,}77 m/s^2,\ \alpha_a = 81{,}37^\circ$

VI.11.



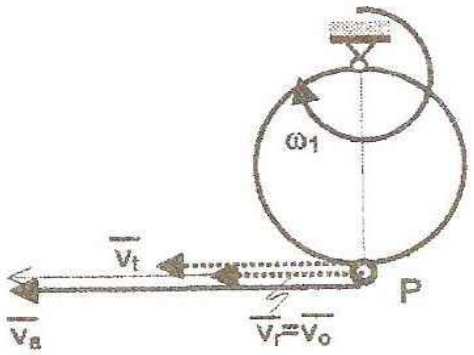
$v_a = 58 cm/s;\ \alpha_v = 58{,}85^\circ\ \ a_a = 45 cm/s^2,\ \alpha_a = 27{,}26^\circ$

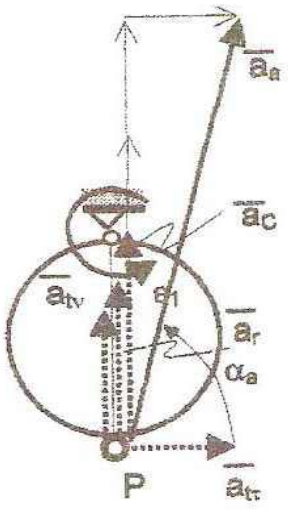
VI.12.



$v_a = 3{,}68 m/s;\ \alpha_v = 106{,}14^\circ\ \ a_a = 6{,}48 m/s^2,\ \alpha_a = 19{,}1^\circ$

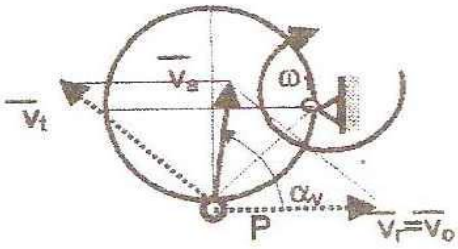
VI.13.

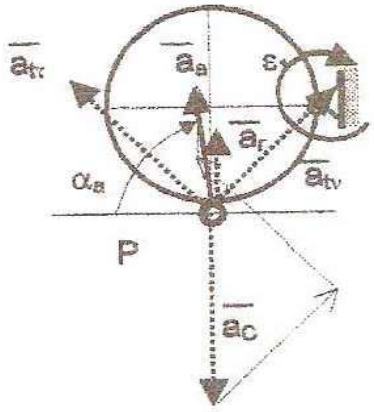




$v_a = 11m/s$; $a_a = 245,17m/s^2$, $\alpha_a = 82,97^o$

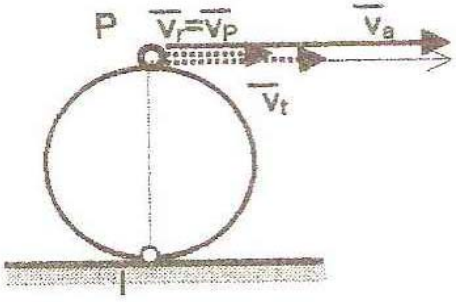
VI.14.

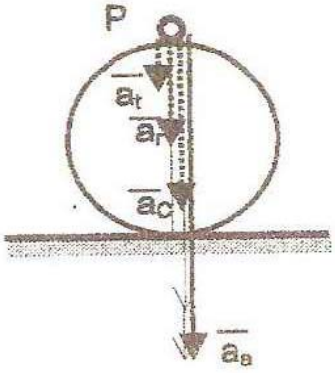




$v_a = 30cm/s$; $\alpha_v = 81,81^o$; $a_a = 100cm/s^2$, $\alpha_a = 89,42^o$

VI.15.

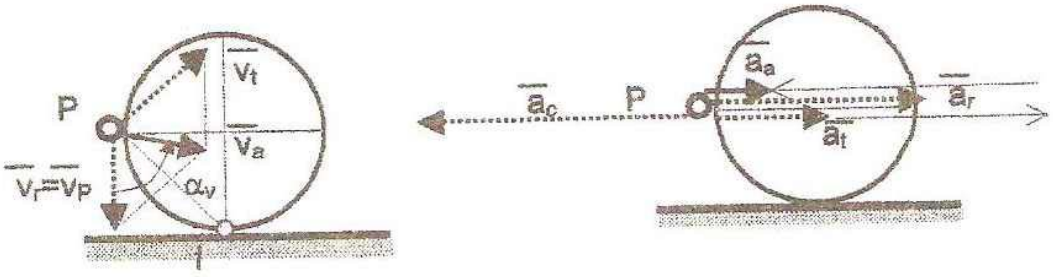




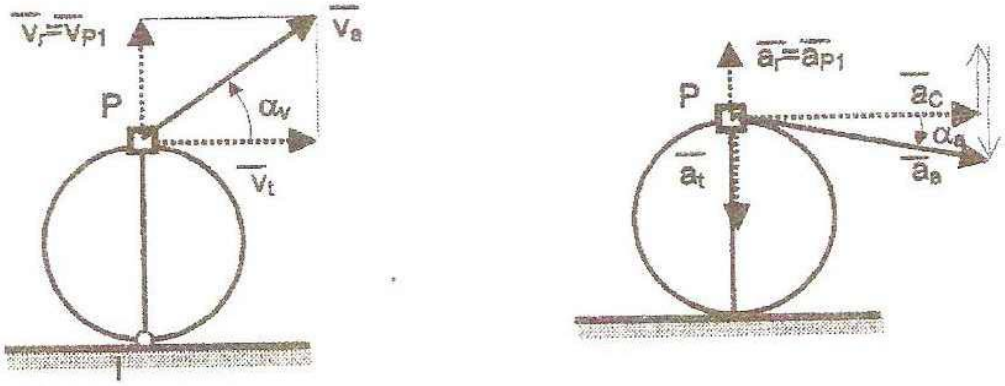
$v_a = 160cm/s$; $a_a = 302,5cm/s^2$

VI.16.

$v_a = 0{,}84 m/s$; $\alpha_v = 81{,}81^\circ$; $a_a = 0{,}34 m/s^2$

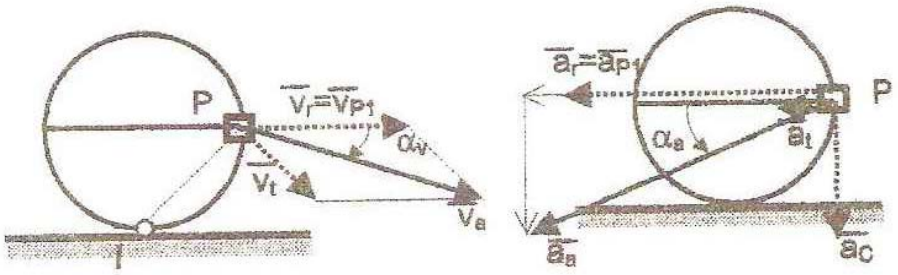


VI.17.



$v_a = 1{,}08 m/s$; $\alpha_v = 33{,}55^\circ$; $a_a = 218 m/s^2$, $\alpha_a = 8{,}18^\circ$

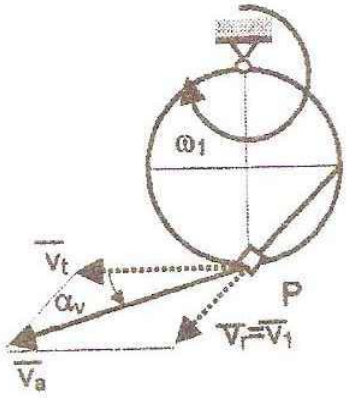
VI.18.

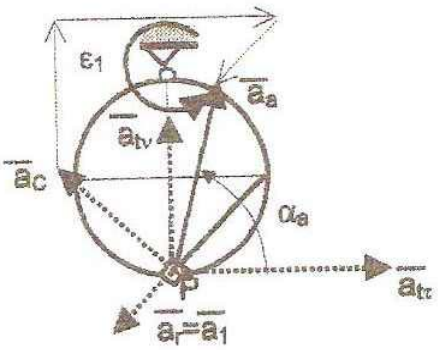


$v_a = 61{,}66$ cm/s, $\alpha_v = 13{,}8^\circ$, $a_a = 69{,}92$ cm/s^2, $\alpha_a = 22{,}5^\circ$.

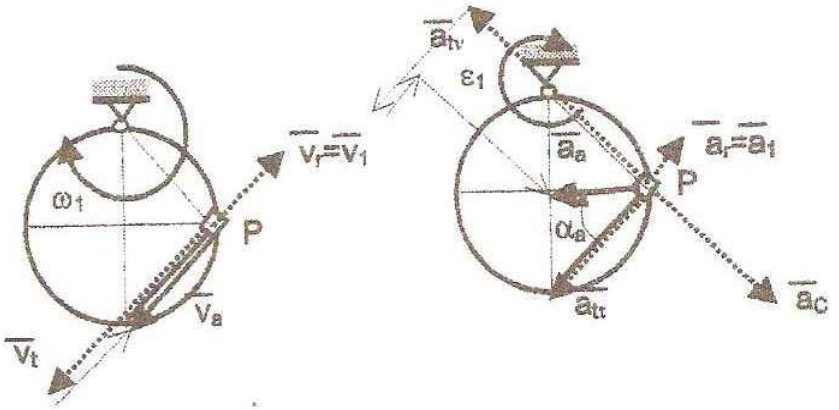
VI.19.

$v_a = 0{,}74 m/s$, $\alpha_v = 22{,}5^\circ$, $a_a = 2{,}35 m/s^2$, $\alpha_a = 78{,}2^\circ$



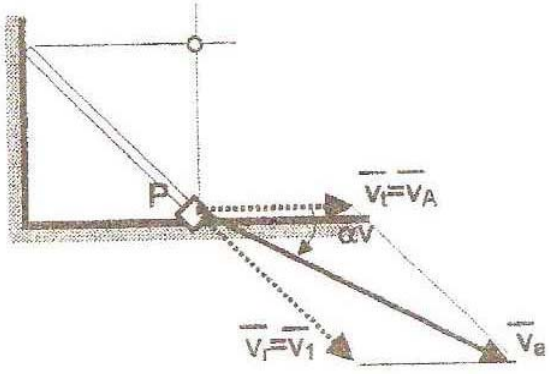


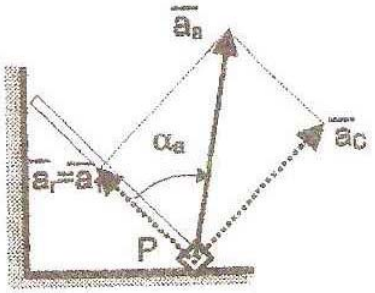
VI.20



$v_a = 1{,}2 m/s$, $a_a = 3{,}06 m/s^2$, $\alpha_a = 43{,}08^o$.

VI.21.

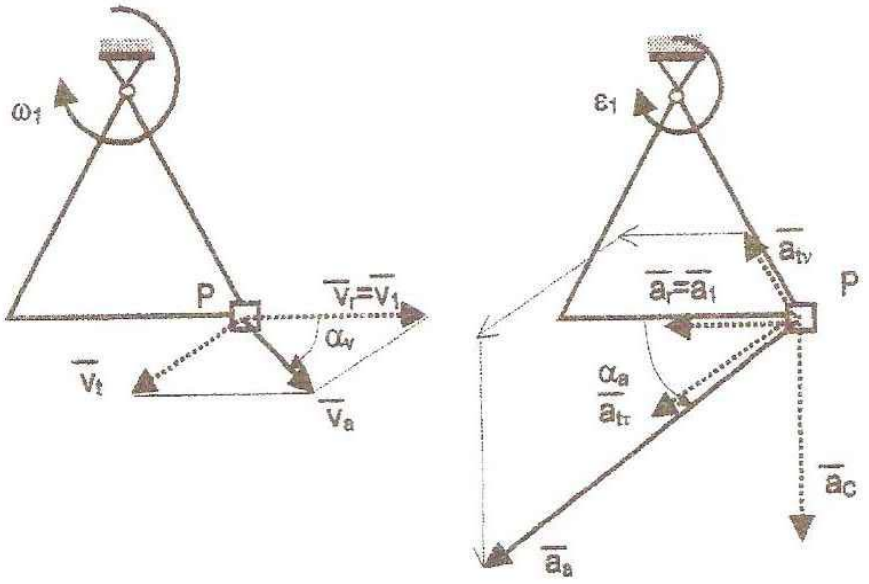




$v_a = 2{,}5 m/s$, $\alpha_v = 34{,}44^o$, $a_a = 5{,}19 m/s^2$, $\alpha_a = 54{,}72^o$

VII.22

$v_a = 41\ cm/s$, $\alpha_v = 47{,}03^o$, $a_a = 3{,}59 m/s^2$, $\alpha_a = 49{,}32^o$



CAPITOLUL VII. PRINCIPIUL LUCRULUI MECANIC VIRTUAL PENTRU STAREA DE REPAUS

VII.1. CALCULUL REACȚIUNILOR LA GRINZILE GERBER

VII.1.1. $V_A = 76{,}67kN \uparrow$; $M_A = 293{,}33\ kNm \circlearrowright$; $V_B = 66{,}67\ kN \uparrow$.

VII.1.2. $V_A = 115daN \downarrow$; $M_A = 500daNm \circlearrowleft$; $V_B = 65daN \uparrow$.

VII.1.3. $V_A = 97{,}5daN \uparrow$; $M_A = 270daNm \circlearrowright$; $V_B = 22{,}5daN \uparrow$.

VII.1.4. $V_A = 480daN \downarrow$; $M_A = 2000daNm \circlearrowleft$; $V_B = 660daN \uparrow$.

VII.1.5. $V_A = 53{,}33KN \downarrow$; $M_B = 230KNm \circlearrowleft$; $V_B = 130KN \uparrow$.

VII.1.6. $V_A = 112KN \uparrow$; $M_B = 16KNm \circlearrowleft$; $V_B = 18KN \uparrow$.

VII.1.7. $V_A = 100KN \uparrow$; $M_B = 150KNm \circlearrowleft$; $V_B = 95KN \uparrow$.

VII.1.8. $V_A = 24{,}85KN \uparrow$; $M_B = 28{,}138\ KNm \circlearrowleft$; $V_B = 14{,}07KN \uparrow$.

VII.1.9. $V_A = 6{,}67daN \uparrow$; $V_B = 13{,}33daNm \uparrow$; $V_C = 90daN \uparrow$.

VII.1.10. $V_A = 100daN \downarrow$; $V_B = 95daNm \uparrow$; $V_C = 20daN \downarrow$.

VII.1.11. $V_A = 353{,}12daN \downarrow$; $V_B = 75{,}94daNm \uparrow$; $V_C = 12{,}81daN \downarrow$.

VII.1.12. $V_A = 104{,}67daN \uparrow$; $V_B = 120{,}83daNm \uparrow$; $V_C = 14daN \uparrow$.

VII.1.13. $V_A = 240daN \downarrow$; $V_B = 170daNm \uparrow$; $V_C = 50daN \downarrow$.

VII.1.14. $V_A = 127,5daN \uparrow$; $V_B = 80daNm \uparrow$; $V_C = 20daN \downarrow$.

VII.1.15. $V_A = 1,25daN \uparrow$; $V_B = 63,75daNm \uparrow$; $V_C = 5,83daN \downarrow$.

VII.1.16. $V_A = 203daN \uparrow$; $V_B = 76,167daNm \uparrow$; $V_C = 22,5daN \downarrow$.

VII.1.17. $V_A = 70daN \uparrow$; $V_B = 5daNm \uparrow$; $V_C = 50daN \uparrow$.

VII.1.18. $V_A = 60daN \uparrow$; $V_B = 37,5daNm \uparrow$; $V_C = 25daN \uparrow$.

VII.1.19. $V_A = 125daN \uparrow$; $V_B = 152daNm \uparrow$; $V_C = 52daN \downarrow$.

VII.1.20. $V_A = 113,33daN \uparrow$; $V_B = 85,33daNm \uparrow$; $V_C = 42daN \downarrow$.

VII.2. CALCULUL REACȚIUNILOR LA CADRE

VII.2.1. $H_A = 78,75kN \rightarrow$; $V_A = 166,87kN \downarrow$; $M_A = 1342,5\ kNm$ ↻ .

VII.2.2. $H_A = 38,75kN \leftarrow$; $V_A = 40kN \uparrow$; $M_A = 240\ kNm$ ↻ .

VII.2.3. $H_A = 45kN \leftarrow$; $V_A = 180\ kN \downarrow$; $M_A = 720kNm$ ↺ .

VII.2.4. $H_A = 9,375kN \leftarrow$; $V_A = 67,5kN \uparrow$; $M_A = 305\ kNm$ ↺ .

VII.2.5. $H_A = 58,33\ kN \rightarrow$; $V_A = 166,25kN \uparrow$; $M_A = 542,5kNm$ ↺ .

VII.2.6. $H_A = 11,67kN \rightarrow$; $V_A = 47,5kN \uparrow$; $M_A = 330\ kNm$ ↺ .

VII.2.7. $H_A = 230daN \leftarrow$; $V_A = 330daN \uparrow$; $M_A = 1020daNm$ ↻ .

VII.2.8. $H_A = 6,25N \rightarrow$; $V_A = 96,67N \uparrow$; $M_A = 13,33Nm$ ↻ .

VII.2.9. $H_A = 83,33kN \rightarrow$; $V_A = 71,11kN \uparrow$; $M_A = 791,11kNm$ ↺ .

VII.2.10. $H_A = 28,75kN \leftarrow$; $V_A = 110kN \uparrow$; $M_A = 330kNm$ ↺ .

VII.2.11. $V_A = 45,56daN \uparrow$; $H_A = 6,67daN \rightarrow$; $V_B = 1,11daN \uparrow$.

VII.2.12. $V_A = 54,16daN \downarrow$; $H_A = 73,61daN \leftarrow$; $V_B = 174,16daN \uparrow$.

VII.2.13. $V_A = 22,5daN \uparrow$; $H_A = 2,5daN \leftarrow$; $V_B = 12,5daN \uparrow$.

VII.2.14. $V_A = 32kN \downarrow$; $H_A = 1,75kN \leftarrow$; $V_B = 142kN \uparrow$.

VII.2.15. $V_A = 11,25kN \uparrow$; $H_A = 42,91kN \leftarrow$; $V_B = 85kN \uparrow$.

VII.2.16. $V_A = 124,64kN \uparrow$; $H_A = 15,24kN \leftarrow$; $V_B = 75,36kN \uparrow$.

VII.2.17. $V_A = 15kN \uparrow$; $H_A = 30kN \rightarrow$; $V_B = 150kN \uparrow$.

VII.2.18. $V_A = 78,75daN \uparrow$; $H_A = 45daN \rightarrow$; $V_B = 26,25daN \downarrow$.

VII.2.19. $V_A = 93,33kN \downarrow$; $H_A = 10kN \rightarrow$; $V_B = 240kN \uparrow$.

VII.2.20. $V_A = 45daN \uparrow$; $H_A = 58,33daN \rightarrow$; $V_B = 10daN \downarrow$.

VII.3. CALCULUL EFORTURILOR ÎN BARELE GRINZILOR CU ZĂBRELE SIMPLE

VII.3.1. $N_1 = -40kN$; $N_2 = -80kN$; $N_3 = 60kN$; $N_4 = 50kN$.

VII.3.2. $N_1 = 0$; $N_2 = -50kN$; $N_3 = -60kN$; $N_4 = 0$.

VII.3.3. $N_1 = -24,02kN$; $N_2 = -20,66kN$; $N_3 = 16,83kN$; $N_4 = -43,25kN$.

VII.3.4. $N_1 = 0$; $N_2 = 8,25kN$; $N_3 = -53,75kN$.

VII.3.5. $N_1 = 0$; $N_2 = 38,75kN$; $N_3 = 4,5kN$; $N_4 = -31kN$.

VII.3.6. $N_1 = -130daN$; $N_2 = -45daN$; $N_3 = -100N$; $N_4 = 62,5daN$.

VII.3.7. $N_1 = -34daN$; $N_2 = -40,86daN$; $N_3 = -8,67daN$; $N_4 = -16daN$.

VII.3.8. $N_1 = -280kN$; $N_2 = 6,73kN$; $N_3 = 297,15kN$; $N_4 = -12,01kN$.

VII.3.9. $N_1 = -120daN$; $N_2 = 60daN$; $N_3 = 36,06daN$.

VII.3.10. $N_1 = 30daN$; $N_2 = 40daN$; $N_3 = -22,37daN$; $N_4 = 10daN$.

VII.3.11. $N_1 = 0$; $N_2 = 80daN$; $N_3 = 37,29daN$; $N_4 = 34,16daN$.

VII.3.12. $N_1 = 96,94daN$; $N_2 = 0$; $N_3 = -80daN$; $N_4 = 28,29daN$.

VII.3.13. $N_1 = 11,87daN$; $N_2 = -13,53daN$; $N_3 = 39,37daN$; $N_4 = -31,25daN$.

VII.3.14. $N_1 = 15daN$; $N_2 = -18,03daN$; $N_3 = -14,14daN$; $N_4 = 60daN$.

VII.3.15. $N_1 = 31,12kN$; $N_2 = -22kN$; $N_3 = 0$; $N_4 = 82,04kN$.

VII.3.16. $N_1 = 28kN$; $N_2 = -60kN$; $N_3 = 0$; $N_4 = 9,62kN$.

VII.3.17. $N_1 = 104,67daN$; $N_2 = -74daN$; $N_3 = 78daN$; $N_4 = -36,78daN$.

VII.3.18. $N_1 = -42,43daN$; $N_2 = 106,66daN$; $N_3 = -113,15daN$; $N_4 = 153,33daN$.

VII.3.19. $N_1 = 13,33kN$; $N_2 = 4,17kN$; $N_3 = -48,89kN$; $N_4 = -82,5kN$.

VII.3.20. $N_1 = -240,37kN$; $N_2 = -3,33kN$; $N_3 = 65kN$; $N_4 = -145,34kN$; $N_5 = -84,65kN$.

CAPITOLUL VIII. STUDIUL MIŞCĂRII UNUI SISTEM MECANIC

VIII.1. FOLOSIREA PRINCIPIULUI LUI D'ALEMBERT PENTRU STUDIUL MIŞCĂRII UNUI SISTEM DE SCRIPEŢI (METODA CINETO STATICĂ)

VIII.1.1. $a_P = 8g/17\uparrow$; $a_C = 4g/17\downarrow$; $\varepsilon_O = 8g/17R$ ↻ ; $\varepsilon_C = 2g/17R$ ↺ ;
$T_1 = 25Mg/17$; $T_2 = 33Mg/17$; $T_3 = 45Mg/17$.

VIII.1.2. $a_P = 5g/11\downarrow$; $a_C = 5g/11\downarrow$; $\varepsilon_O = 10g/11R$ ↻ ; $\varepsilon_C = 5g/11R$ ↺ ;
$T_1 = 10Mg/11$; $T_2 = 6Mg/11$; $T_3 = 20Mg/11$.

VIII.1.3. $a_P = 14g/29\uparrow$; $a_C = 14g/29\downarrow$; $\varepsilon_O = 14g/29R$ ↻ ; $\varepsilon_C = 7g/29R$ ↺ ;
$T_1 = 43Mg/29$; $T_2 = 64Mg/29$; $T_3 = 56Mg/29$.

VIII.1.4. $a_P = 12g/13\downarrow$; $a_C = 6g/13\downarrow$; $\varepsilon_O = 3g/13R$ ↻ ; $\varepsilon_C = 6g/13R$ ↺ ;
$T_1 = Mg/13$; $T_2 = 18Mg/13$; $T_3 = 11Mg/13$.

VIII.1.5. $a_P = 9g/17\downarrow$; $a_C = 9g/17\downarrow$; $\varepsilon_O = 18g/17R$ ↺ ; $\varepsilon_C = 9g/17R$ ↺ ;
$T_1 = 18Mg/17$; $T_2 = 8Mg/17$; $T_3 = 54Mg/17$.

VIII.1.6. $a_P = g/2\uparrow$; $a_Q = g/2\downarrow$; $a_S = g/2\downarrow$; $\varepsilon_O = g/2R$ ↺ ;
$T_1 = 3Mg/2$; $T_2 = 5Mg/2$; $T_3 = 3Mg/2$.

VIII.1.7. $a_P = 3g/5\downarrow$; $a_C = 6g/5\downarrow$; $\varepsilon_C = 3g/5R$ ↺ ;
$T_1 = 8Mg/5$; $T_2 = 2Mg/5$; $T_3 = -2Mg/5$.

VIII.1.8. $a_{C1} = 2g/19\uparrow$; $a_{C2} = 2g/19\downarrow$; $\varepsilon_O = 4g/19R$ ↻ ; $\varepsilon_{C1} = 2g/19R$ ↻ ;
$\varepsilon_{C2} = 2g/19R$ ↺ ; $T_1 = 40Mg/19$; $T_2 = 44Mg/19$; $T_3 = 48Mg/19$;
$T_4 = 54Mg/19$.

VIII.1.9. $a_C = 5g/18\downarrow$; $\varepsilon_O = 5g/6R$ ↺ ; $\varepsilon_C = 5g/18R$ ↺ ;
$T_1 = 115Mg/18$; $T_2 = 5Mg/6$.

VIII.1.10. $a_C = 16g/27 \downarrow$; $\varepsilon_O = 8g/9R \curvearrowleft$; $\varepsilon_C = 8g/27R \curvearrowright$;
$T_1 = 64Mg/27$; $T_2 = 8Mg/9$.

VIII.1.11. $a_P = 93g/95 \downarrow$; $a_Q = 124g/95 \downarrow$; $a_C = 62g/95 \downarrow$; $\varepsilon_C = 31g/95R \curvearrowright$;
$T_1 = 274Mg/95$; $T_2 = 2Mg/95$; $T_3 = -58Mg/95$.

VIII.1.12. $a_P = 20g/53 \downarrow$; $a_Q = 60g/53 \downarrow$; $a_C = 20g/53 \downarrow$; $\varepsilon_C = 20g/53R \curvearrowright$;
$T_1 = 236Mg/53$; $T_2 = 66Mg/53$; $T_3 = -28Mg/53$.

VIII.1.13. $a_P = 3g/13 \uparrow$; $a_Q = 6g/13 \downarrow$; ; $a_S = 6g/13 \downarrow$; $\varepsilon_O = 3g/13R \curvearrowright$;
$T_1 = 16Mg/13$; $T_2 = 35Mg/13$; $T_3 = 21Mg/13$.

VIII.1.14. $a_P = 3g/20 \uparrow$; $a_C = 3g/20 \downarrow$; $\varepsilon_O = 3g/20R \curvearrowright$; $\varepsilon_C = 3g/20R \curvearrowleft$;
$T_1 = 23Mg/20$; $T_2 = 31Mg/20$; $T_3 = 37Mg/20$.

VIII.1.15. $a_P = 44g/67 \downarrow$; $a_C = 44g/67 \downarrow$; $\varepsilon_O = 66g/67R \curvearrowleft$; $\varepsilon_C = 22g/67R \curvearrowright$;
$T_1 = 187Mg/67$; $T_2 = 66Mg/67$; $T_3 = 23Mg/67$.

VIII.1.16. $a_P = 14g/29 \downarrow$; $a_C = 7g/29 \downarrow$; $\varepsilon_O = 21g/29R \curvearrowright$; $\varepsilon_C = 7g/29R \curvearrowright$;
$T_1 = 129Mg/29$; $T_2 = 63Mg/29$; $T_3 = 60Mg/29$.

VIII.1.17. $a_P = 27g/32 \downarrow$; $a_C = 9g/32 \downarrow$; $\varepsilon_O = 9g/16R \curvearrowleft$; $\varepsilon_C = 9g/32R \curvearrowright$;
$T_1 = 107Mg/32$; $T_2 = 9Mg/8$; $T_3 = 5Mg/32$.

VIII.1.18. $a_P = 20g/61 \downarrow$; $a_C = 40g/61 \downarrow$; $\varepsilon_O = 60g/61R \curvearrowright$; $\varepsilon_C = 20g/61R \curvearrowright$;
$T_1 = 120Mg/61$; $T_2 = 164Mg/61$; $T_3 = 212Mg/61$.

VIII.1.19. $a_P = 5g/7 \downarrow$; $a_C = 5g/14 \downarrow$; $\varepsilon_O = 15g/14R \curvearrowright$; $\varepsilon_C = 5g/14R \curvearrowleft$;
$T_1 = 15Mg/14$; $T_2 = 2Mg/7$; $T_3 = 61Mg/14$.

VIII.1.20. $a_P = 56g/127 \downarrow$; $a_C = 14g/127 \uparrow$; $\varepsilon_O = 28g/127R \curvearrowright$;
$\varepsilon_C = 14g/127R \curvearrowright$; $T_1 = 284Mg/127$; $T_2 = 148Mg/127$;
$T_3 = 134Mg/127$.

VIII.1.21. $a_P = 4(3-\mu)g/17 \leftarrow$; $a_C = 2(3-\mu)g/17 \downarrow$; $\varepsilon_O = 4(3-\mu)g/17R \curvearrowleft$;
$\varepsilon_C = (3-\mu)g/17R \curvearrowright$; $T_1 = 3(14+\mu)Mg/17$; $T_2 = 3(8+3\mu)Mg/17$;
$T_3 = (12+13\mu)Mg/17$.

VIII.1.22. $a_P = 2g/7 \downarrow$; $a_C = g/7 \leftarrow$; $\varepsilon_O = 2g/7R \curvearrowleft$; $\varepsilon_C = g/7R \curvearrowleft$;
$T_1 = 5Mg/7$; $T_2 = 3Mg/7$.

VIII.1.23. $a_P = 2(2-3\mu)\ g/21 \downarrow$; $a_S = (2-3\mu)\ g/21 \leftarrow$; $a_Q = (2-3\mu)\ g/21 \leftarrow$;
$\varepsilon_O = (2-3\mu)\ g/21R \curvearrowleft$; $T_1 = (17+6\mu)\ Mg/21$; $T_2 = (6+54\mu)Mg/21$;

$T_3 = 2(2+18\mu)\ Mg/21$.

VIII.1.24. $a_P = 8(9-2\mu)\ g/165$ ← ; $a_C = 2(9-2\mu)\ g/55$ ↓ ; $\varepsilon_O = 4(9-2\mu)\ g/165R$ ↶ ;
$\varepsilon_C = 2(9-2\mu)\ g/55R$ ↷ ; $T_1 = (138+6\mu)\ Mg/55$; $T_2 = (84+18\mu)\ Mg/55$;
$T_3 = (72+149\mu)\ Mg/165$.

VIII.1.25. $a_P = 16g/75$ ↓ ; $a_C = 4g/75$ ← ; $\varepsilon_O = 8g/75R$ ↶ ; $\varepsilon_C = 4g/75R$ ↶ ;
$T_1 = 59Mg/75$; $T_2 = 2Mg/25$.

VIII.1.26. $a_P = 3g/10$ ↓ ; $a_C = g/10$ ← ; $\varepsilon_O = 3g/10R$ ↶ ; $\varepsilon_C = g/10R$ ↶ ;
$T_1 = 7Mg/10$; $T_2 = 2Mg/5$.

VIII.1.27. $a_P = (10-3\mu)g/14$ ← ; $a_C = (10-3\mu)g/42$ ↓ ; $\varepsilon_O = (10-3\mu)g/14R$ ↶ ;
$\varepsilon_C = (10-3\mu)g/42R$ ↷ ; $T_1 = (130+3\mu)Mg/21$; $T_2 = (10+4\mu)Mg/7$;
$T_3 = (10+11\mu)Mg/14$.

VIII.1.28. $a_P = 3g/22$ ↓ ; $a_C = g/11$ ← ; $\varepsilon_O = 3g/22R$ ↶ ; $\varepsilon_C = g/22R$ ↶ ;
$T_1 = Mg/11$; $T_2 = 5Mg/11$.

VIII.1.29. $a_P = (8-\mu)g/15$ ← ; $a_C = (8-\mu)g/15$ ↓ ; $\varepsilon_O = (8-\mu)g/15\ R$ ↶ ;
$\varepsilon_C = (8-\mu)g/45\ R$ ↷ ; $T_1 = 4(8-\mu)Mg/15$; $T_2 = (8+4\mu)Mg/5$;
$T_3 = 45(8+4\mu)Mg/15$.

VIII.1.30. $a_P = 7(32-7\mu)g/269$ ← ; $a_C = 4(32-7\mu)g/269$ ↓ ;
$\varepsilon_O = 7(32-7\mu)g/538R$ ↶ ; $\varepsilon_C = (32-7\mu)g/269R$ ↷ ;
$T_1 = (680+53\mu)Mg/269$; $T_2 = (448+171\mu)Mg/269$;
$T_3 = (224+22\mu)Mg/269$.

VIII.2. STUDIUL MIŞCĂRII UNUI SISTEM FOLOSIND ECUAŢIILE LAGRANGE DE SPEŢA A II-A SAU TEOREMA ENERGIEI CINETICE

VIII.2.1. $$\begin{cases} \ddot{\theta}(2r^2 + 8l^2) + 4r\dot{r}\dot{\theta} + g(r-l)\sin\theta = 0 \\ 2\ddot{r} - 2r\dot{\theta}^2 - g\cos\theta = 0 \end{cases}$$

VIII.2.2. $$\begin{cases} 2\ddot{\theta} + \ddot{\varphi}\cos(\theta-\varphi) + \dot{\varphi}^2\sin(\theta-\varphi) + 2g\sin\theta\ //\ l = 0 \\ \ddot{\varphi} + \ddot{\theta}\cos(\theta-\varphi) - \dot{\theta}^2\sin(\theta-\varphi) + g\sin\varphi\ //\ l = 0 \end{cases}$$

VIII.2.3. $$\begin{cases} 6\ddot{\theta} + \ddot{\varphi}\cos(\theta-\varphi) + \dot{\varphi}^2\sin(\theta-\varphi) + 4g\sin\theta\ //\ l = 0 \\ \ddot{\varphi} + 2\ddot{\theta}\cos(\theta-\varphi) - 2\dot{\theta}^2\sin(\theta-\varphi) + g\sin\varphi\ //\ l = 0 \end{cases}$$

VIII.2.4. $$\begin{cases} \ddot{\theta} + \ddot{\varphi}\cos(\theta-\varphi) + \dot{\varphi}^2\sin(\theta-\varphi) + g\sin\theta\,/\!/ = 0 \\ 4\ddot{\varphi} + 3\ddot{\theta}\cos(\theta-\varphi) - 3\dot{\theta}^2\sin(\theta-\varphi) + 3g\sin\varphi\,/\!/ = 0 \end{cases}$$

VIII.2.5. $$\begin{cases} 30\ddot{\theta} + 5\ddot{\varphi}\cos(\theta-\varphi) + 5\dot{\varphi}^2\sin(\theta-\varphi) + 7g\sin\theta\,/\!/ = 0 \\ 5\ddot{\varphi} + 10\ddot{\theta}\cos(\theta-\varphi) - 10\dot{\theta}^2\sin(\theta-\varphi) + g\sin\varphi\,/\!/ = 0 \end{cases}$$

VIII.2.6. $$\begin{cases} 28\ddot{\theta} + 12\ddot{\varphi}\cos(\theta-\varphi) + 12\dot{\varphi}^2\sin(\theta-\varphi) + 15g\sin\theta\,/\!/ = 0 \\ 4\ddot{\varphi} + 6\ddot{\theta}\cos(\theta-\varphi) - 6\dot{\theta}^2\sin(\theta-\varphi) + 3g\sin\varphi\,/\!/ = 0 \end{cases}$$

VIII.2.7. $$\begin{cases} 4\ddot{\theta} + \ddot{\varphi}\cos(\theta-\varphi) + \dot{\varphi}^2\sin(\theta-\varphi) + g\sin\theta\,/R = 0 \\ 3\ddot{\varphi} + 8\ddot{\theta}\cos(\theta-\varphi) - 8\dot{\theta}^2\sin(\theta-\varphi) + 2g\sin\varphi\,/R = 0 \end{cases}$$

VIII.2.8. $$\begin{cases} 40\ddot{\theta} + 8\ddot{\varphi}\cos(\theta-\varphi) + 8\dot{\varphi}^2\sin(\theta-\varphi) + 11g\sin\theta\,/R = 0 \\ 3\ddot{\varphi} + 8\ddot{\theta}\cos(\theta-\varphi) - 8\dot{\theta}^2\sin(\theta-\varphi) + 2g\sin\varphi\,/R = 0 \end{cases}$$

VIII.2.9. $$\begin{cases} 27\ddot{\theta} + 24\ddot{\varphi}\cos(\theta-\varphi) + 24\dot{\varphi}^2\sin(\theta-\varphi) + 14g\sin\theta\,/R = 0 \\ 8\ddot{\varphi} + 6\ddot{\theta}\cos(\theta-\varphi) - 6\dot{\theta}^2\sin(\theta-\varphi) + 3g\sin\varphi\,/R = 0 \end{cases}$$

VIII.2.10. $$\begin{cases} \ddot{r} + \ddot{\theta}R^2\cos\theta - \dot{\theta}^2R^2\sin\theta = 0 \\ 8R\ddot{\theta} + 3R\ddot{r}\cos + 3g\sin\theta = 0 \end{cases}$$

VIII.2.11. $$\begin{cases} 2\ddot{r} - 2r\dot{\theta}^2 - R\dot{\theta}^2 - 2g\cos\theta = 0 \\ r^2\ddot{\theta} + 7R^2\ddot{\theta} + R\dot{r}\dot{\theta} + Rr\ddot{\theta} + 2{,}5gR\sin\theta + gr\sin\theta = 0 \end{cases}$$

VIII.2.12. $$\begin{cases} 7\ddot{r} + R\ddot{\theta}\cos\theta - R\dot{\theta}^2\sin\theta = 0 \\ R\ddot{\theta} + \ddot{r}\cos\theta + g\sin\theta = 0 \end{cases}$$

VIII.2.13. $$\begin{cases} 7\ddot{r} + R\ddot{\theta}\cos\theta - R\dot{\theta}^2\sin\theta = 0 \\ R\ddot{\theta} + \ddot{r}\cos\theta + g\sin\theta = 0 \end{cases}$$

VIII.2.14. $$\begin{cases} 4l^2\ddot{\theta} + 2r\dot{r}\dot{\theta} + r^2\ddot{\theta}^2 - 4l\dot{r}\dot{\theta}\sin^2\theta + 4(l-r)\ddot{\theta}\sin^2\theta + \\ + 4l(l-r)\dot{\theta}^2\sin\theta\cos\theta + 2l\dot{r}\sin\theta\cos\theta - 5gl\sin\theta + gr\sin\theta = 0 \\ \ddot{r} + 2l\ddot{\theta}\sin\theta\cos\theta + 2l\dot{\theta}^2\cos^2\theta - r\dot{\theta}^2 - g\cos\theta = 0 \end{cases}$$

VIII.2.15. $$\begin{cases} \ddot{r} + l\ddot{\theta}\cos\theta - l\dot{\theta}^2\sin\theta = 0 \\ 5l\ddot{\theta} + \ddot{r}\cos\theta + 5g\sin\theta = 0 \end{cases}$$

VIII.2.16. $\begin{cases} \ddot{\theta} = 0 \\ R\ddot{\varphi} + g\sin\varphi = 0 \end{cases}$

VIII.2.17. $\begin{cases} \ddot{r} + 2l\ddot{\theta}\cos\theta - 2l\dot{\theta}^2\sin\theta = 0 \\ 14l\ddot{\theta} + \ddot{r}\cos\theta + 8g\sin\theta = 0 \end{cases}$

VIII.2.18. $\begin{cases} 4\ddot{\theta} + 3\ddot{\varphi}\cos(\theta - \varphi) + 3\dot{\varphi}^2\sin(\theta - \varphi) + g\sin\theta / R = 0 \\ 3\ddot{\varphi} + \ddot{\theta}\cos(\theta - \varphi) - \dot{\theta}^2\sin(\theta - \varphi) + g\sin\varphi / R = 0 \end{cases}$

VIII.2.19. $\begin{cases} 6M\ddot{r} + 3k\,r - kl\,\theta = 0 \\ 2Ml\ddot{\theta} - k\,r + 3kl\,\theta = 0 \end{cases}$

VIII.2.20. $\begin{cases} 4Ml\ddot{\theta} + (3Mg + 4kl)\,\theta + 4kl\,\varphi = 0 \\ 4Ml\ddot{\varphi} + 2kl\,\theta + (3Mg + 2kl)\,\varphi = 0 \end{cases}$

VIII.2.21. $\begin{cases} 4Ma\ddot{\theta} + (Mg + 9ka)\,\theta + 6ka\,\varphi = 0 \\ 4Ma\ddot{\varphi} + 36ka\,\theta + (Mg + 28ka)\,\varphi = 0 \end{cases}$

VIII.2.22. $\begin{cases} 4\ddot{r} + 3l\ddot{\theta}\cos\theta - 3l\dot{\theta}^2\sin\theta = 0 \\ 4l\ddot{\theta} + 3\ddot{r}\cos\theta + 3g\sin\theta = 0 \end{cases}$

VIII.2.23. $\begin{cases} 4\ddot{r} - 3l\ddot{\theta}\sin\theta - 3l\dot{\theta}^2\cos\theta - 4g = 0 \\ 4l\ddot{\theta} - r\sin\theta + 3g\cos\theta = 0 \end{cases}$

VIII.2.24. $64l\ddot{\theta}\sin^2\theta + 64l\dot{\theta}^2\sin\theta\cos\theta + 10l\ddot{\theta} - 9g\cos\theta = 0$

VIII.2.25. $\ddot{\theta} = 0$

VIII.2.26. $10R\ddot{\theta} + 28R\ddot{\theta}\sin^2\theta + 28R\dot{\theta}^2\sin\theta\cos\theta + 3g\cos\theta = 0$

VIII.2.27. $6l\ddot{\theta} + 2l\,\theta\sin^2\ddot{\theta} + 2l\dot{\theta}^2\sin\theta\cos\theta + 5g\cos\theta = 0$

VIII.2.28. $\begin{cases} \ddot{\theta}(l_o + r)^2 + 2\ddot{\theta}\dot{r}(l_o + r) + g(l_o + r)\sin\theta = 0 \\ M\ddot{r} - M\dot{\theta}^2(l_o + r) + k\,r - Mg\cos\theta = 0 \end{cases}$

VIII.2.29. $\ddot{\theta}(l_o + R\,\theta)^2 + \dot{\theta}^2(l_o + R\,\theta)R + 4R^2\ddot{\theta}\cos^2\theta - 6R^2\dot{\theta}^2\sin\theta\cos\theta -$
$- 4\ddot{\theta}(l_o + R\,\theta)R\sin\theta\cos\theta - 2R\dot{\theta}^2(l_o + R\,\theta)\cos^2\theta +$
$+ 2R\dot{\theta}^2(l_o + R\,\theta)\sin^2\theta - 2gR\cos\theta + g(l_o + R\,\theta)\sin\theta = 0$

VIII.2.30. $10MR\ddot{\theta} + (3Mg + 3kR)\,\theta = 0$